10. Use the rules of logarithms to simplify the following and then find $\frac{dy}{dx}$ of each function.

(i) $y = \log_e(3x + 1)^3$ (ii) $y = \log_e\left(\frac{2x + 1}{1 - 3x}\right)$ (iii) $y = \log_e\sqrt{1 + x^2}$

Rewrite to simplify

$\log x^n = n \log x$

$f(x) \to f'(x)$

$\ln x \to \frac{1}{x}$

$$y = \ln(3x+1)^3 = 3\ln(3x+1)$$

$$\frac{dy}{dx} = 3\underbrace{\left(\frac{1}{3x+1}\right)}\cdot\underbrace{\left(3\right)}$$

$$= \frac{9}{3x+1}$$

10. Use the rules of logarithms to simplify the following and then find $\frac{dy}{dx}$ of each function.

(iv) $y = \log_e\sqrt{\sin x}$ (v) $y = \log_e(x^2 + 4)^2$ (vi) $y = \log_e\sqrt{\frac{x}{1 + x}}$

Rewrite to suit

$\log \frac{a}{b} = \log a - \log b$

$\log a^n = n \log a$

$f(x) \to f'(x)$

$\ln x \to \frac{1}{x}$

$$y = \ln\sqrt{\frac{x}{1+x}} = \ln\sqrt{x} - \ln\sqrt{1+x}$$

$$= \ln x^{\frac{1}{2}} - \ln(1+x)^{\frac{1}{2}}$$

$$= \frac{1}{2}\ln x - \frac{1}{2}\ln(1+x)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x}\right) - \frac{1}{2}\left[\left(\frac{1}{1+x}\right)(1)\right]$$

$$= \frac{1}{2}\left[\frac{1}{x} - \frac{1}{1+x}\right]$$

$$= \frac{1}{2}\left[\frac{(1+\cancel{x}) - (\cancel{x})}{x(1+x)}\right]$$

$$= \frac{1}{2}\left(\frac{1}{x(1+x)}\right) = \frac{1}{2x(1+x)}$$

10. Use the rules of logarithms to simplify the following and then find $\frac{dy}{dx}$ of each function.

(iv) $y = \log_e \sqrt{\sin x}$ (v) $y = \log_e (x^2 + 4)^2$ (vi) $y = \log_e \sqrt{\frac{x}{1+x}}$

Rewrite to suit

$\ln a^n = n \ln a$

CHAIN RULE

$f(x) \rightarrow f'(x)$

$\ln x \rightarrow \frac{1}{x}$

$\sin x \rightarrow \cos x$

$y = \ln \sqrt{\sin x} = \frac{1}{2} \ln(\sin x)$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{\sin x}\right)(\cos x)$$

$$= \frac{\cos x}{2 \sin x}$$

$$= \frac{1}{2} \cot x$$

11. If $y = \ln 3x^4$, find $\frac{d^2y}{dx^2}$.