3. Simultaneous equations in context

Example 4

An opera was attended by 240 people. Two ticket prices, €31 and €16, were available. If the total takings on the night were €5595, find using this data

(i) two linear equations connecting the two types of tickets sold
(ii) the number of €31 tickets sold
(iii) the number of €16 tickets sold.

let x = no. of €31 tickets sold
let y = no. of €16 tickets sold

$x + y = 240$
$31x + 16y = 5595$

etc..

Example 5

Fifty, twenty and ten cent coins are collected from a coin machine and counted. The total value of the coins is €32. When counting, the cashier noted that twice the number of twenty cent coins, added to the number of ten cent coins, equalled three times the number of fifty cent coins. She then noticed that four times the number of fifty cent coins, added to the number of ten cent coins, equalled six times the number of twenty cent coins.
Find the number of each type of coin in the machine.

let x = the no. of 50c coins
let y = the no. of 20c coins
let z = the no. of 10c coins

$0.5x + 0.2y + 0.1z = 32$
$5x + 2y + z = 320$

$2y + z = 3x \Rightarrow 3x - 2y - z = 0$

$4x + z = 6y \Rightarrow 4x - 6y + z = 0$

etc ...

5. Solve the following equations with three unknowns.

(i) $2x + y + z = 8$ ①
$5x - 3y + 2z = 3$ ②
$7x + y + 3z = 20$ ③

eliminate z

② $5x - 3y + 2z = 3$
$-2$① $-4x - 2y - 2z = -16$
$x - 5y = -13$ ④

③ $7x + y + 3z = 20$
$-3$① $-6x - 3y - 3z = -24$
$x - 2y = -4$ ⑤

Solve for x and y

④ $x - 5y = -13$
$-$⑤ $-x + 2y = 4$
$-3y = -9$ $\Rightarrow$ $y = 3$

Sub into ⑤ $x - 2(3) = -4 \Rightarrow x - 6 = -4 \Rightarrow x = 2$

Sub into ① $2(2) + 3 + z = 8$
$4 + 3 + z = 8$
$7 + z = 8$ $\Rightarrow$ $z = 1$

Answer pt $(2, 3, 1)$

5. Solve the following equations with three unknowns.

(ii) $2x - y - z = 6$ ①
$3x + 2y + 3z = 3$ ②
$4x + y - 2z = 3$ ③

eliminate y

② $3x + 2y + 3z = 3$
$2$① $4x - 2y - 2z = 12$
$7x + z = 15$ ④

③ $4x + y - 2z = 3$
$2x - y - z = 6$
$6x - 3z = 9$ ⑤

eliminate z

$3$④ $21x + 3z = 45$
⑤ $6x - 3z = 9$
$27x = 54$ $\Rightarrow$ $x = \frac{54}{27}$ $\Rightarrow$ $x = 2$

sub into ④ $7(2) + z = 15$
$14 + z = 15$ $\Rightarrow$ $z = 1$

sub into ③ $4(2) + y - 2 = 3$
$8 + y - 2 = 3$
$6 + y = 3$ $\Rightarrow$ $y = -3$

pt. $(2, -3, 1)$

5. Solve the following equations with three unknowns.

(iii) $2x + y - z = 9$ (1)
$x + 2y + z = 6$ (2)
$3x - y + 2z = 17$ (3)

eliminate z

(2) $x + 2y + z = 6$
(1) $2x + y - z = 9$
$3x + 3y = 15$
$x + y = 5$ (4)

(3) $3x - y + 2z = 17$
2(1) $4x + 2y - 2z = 18$
$7x + y = 35$ (5)

eliminate y

(5) $7x + y = 35$
−(4) $-x - y = -5$
$6x = 30$ $\Rightarrow x = 5$

sub into (4) $5 + y = 5$ $\rightarrow y = 0$

sub into (2) $5 + 2(0) + z = 6$
$5 + z = 6$ $\Rightarrow z = 1$

Solution pt. $(5, 0, 1)$

6. Find the point of intersection of each of the following sets of planes.

(i) $2a + b + c = 8$ (1)
$5a - 3b + 2c = -3$ (2)
$7a - 3b + 3c = 1$ (3)

eliminate b

(3) $7a - 3b + 3c = 1$
−(2) $-5a + 3b - 2c = 3$
$2a + c = 4$ (4)

3(1) $6a + 3b + 3c = 24$
+(3) $+7a - 3b + 3c = 1$
$13a + 6c = 25$ (5)

eliminate c

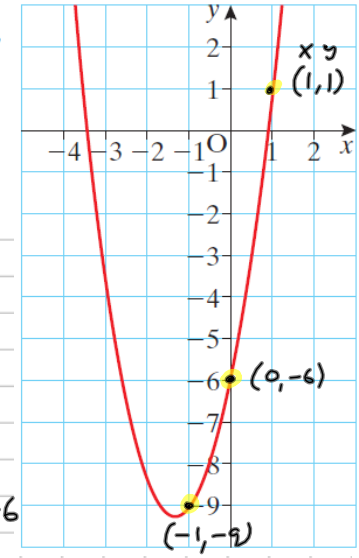
(5) $13a + 6c = 25$
−6(4) $-12a - 6c = -24$
$a = 1$

sub into (4) $2(1) + c = 4$ $\Rightarrow c = 2$

sub into (1) $2(1) + b + 2 = 8$
$b + 4 = 8$ $\Rightarrow b = 4$

Solution pt $(a, b, c) = (1, 4, 2)$

9. A curve of the form $f(x) = y = ax^2 + bx + c$ is drawn as shown.
By using any three points on the curve, form three equations connecting the coefficients a, b and c and hence solve to find $f(x)$.



Sub in $(-1,-9)$

$-9 = a(-1)^2 + b(-1) + c$

$-9 = a - b + c$

$a - b + c = -9$ (1)

Sub in $(1,1)$

$a(1)^2 + b(1) + c = 1$

$a + b + c = 1$ (2)

Sub in $(0,-6)$

$a(0)^2 + b(0) + c = -6$

$c = -6$ (3)

(2) $a + b + c = 1$

+(1) $a - b + c = -9$

$2a + 2c = -8 \Rightarrow a + c = -4$ (4)

Sub in (3) $\rightarrow a - 6 = -4 \rightarrow a = 2$

Sub into (2) $2 + b - 6 = 1 \Rightarrow b - 4 = 1 \Rightarrow b = 5$

$f(x) = 2x^2 + 5x - 6$