Algebraic identities and factors

Example 4

Given that $(x - t)^2$ is a factor of $x^3 + 3px + c$, show that $p = -t^2$ and $c = 2t^3$.

Factors divide in evenly

$(a+b)^2 = a^2 + 2ab + b^2$

$(x-t)^2 = x^2 - 2tx + t^2$

divide

$$
\begin{array}{r|l}
 & x + 2t \\
x^2 - 2tx + t^2 & x^3 + 0x^2 + 3px + c \\
 & \mp x^3 \pm 2tx^2 \mp t^2x \\
 & 2tx^2 + (3p - t^2)x + c \\
 & \mp 2tx^2 \pm 4t^2x \mp 2t^3 \\
 & 0x^2 \quad 0x \quad 0
\end{array}
$$

$\Rightarrow$

$3p - t^2 + 4t^2 = 0$

$3p + 3t^2 = 0$

$\div 3$ $\quad p + t^2 = 0$

$p = -t^2$ ✓ QED

$c - 2t^3 = 0$

$c = 2t^3$ ✓

17. If $\frac{1}{(x+1)(x-1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$ for all x, find values for A and B.

LHS RHS

multiply by LCD

$1 = (x-1)A + (x+1)B$

expand

$1 = Ax - A + Bx + B$

$0x + 1 = (A+B)x + (B-A)$

Solve 2 equation

$A + B = 0$

$A = -B$

$B - A = 1$

$B - -B = 1$

$2B = 1$

$B = \frac{1}{2}$

$A = -\frac{1}{2}$

20. If $(x-3)^2$ is a factor of x^3+ax+b, find the value of a and the value of b.

factors divide evenly!

$(a+b)^2 = a^2+2ab+b^2$

$(x-3)^2 = x^2-6x+9$

divide

$$
\begin{array}{r|l}
 & x + 6 \\
x^2-6x+9 & \cancel{x^3} + 0x^2 + ax + b \\
 & \mp\cancel{x^3} \pm 6x^2 \mp 9x \\
 & \qquad \cancel{6x^2} + (a-9)x + b \\
 & \qquad \mp\cancel{6x^2} \pm 36x \mp 54 \\
 & \qquad\qquad 0x + 0
\end{array}
$$

Conclude

① $a-9+36=0$

$a+27=0$

$a=-27$

② $b-54=0$

$b=54$