Example 2

Simplify $\dfrac{x^2 - 9y^2}{3x + 9y}$

FACTORISE

$$\frac{(x+3y)(x-3y)}{3(x+3y)} = \frac{x-3y}{3}$$

Factors and solutions are related

solve $x^2 + 2x + 1 = 0$

factors → $(x+1)(x+1) = 0$

$x+1=0$ | $x+1=0$

$x=-1$ | $x=-1$

solns
roots

Example 3

Factorise (i) $3x^2 + 10x + 8$ (ii) $x^2 - 2\sqrt{2}x - 6$

(ii) Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$
$b = -2\sqrt{2}$
$c = -6$

$$x = \frac{+2\sqrt{2} \pm \sqrt{(2\sqrt{2})^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{2\sqrt{2} \pm \sqrt{32}}{2} = \frac{2\sqrt{2} \pm 4\sqrt{2}}{2}$$

$x = \sqrt{2} \pm 2\sqrt{2}$

$x = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$ ✓

or $x = \sqrt{2} - 2\sqrt{2} = -\sqrt{2}$

⇒ Factors are:

$(x - 3\sqrt{2})(x + \sqrt{2})$

Exercise 1.3

Using the highest common factor, factorise each of the following:

1. $5x^2 - 10x$ **2.** $6ab - 12bc$ **3.** $3x^2 - 6xy$

① HCF $5x$ ⇒ $5x(x-2)$

② HCF $6b$ ⇒ $6b(a-2c)$

③ HCF $3x$ ⇒ $3x(x-2y)$

Factorise each of the following by grouping terms.

12. $2c^2 - 4cd + c - 2d$

$= c(2c+1) - 2d(2c+1)$ ✓

$= (c - 2d)(2c+1)$

13. $8ax + 4ay - 6bx - 3by$

$4a(2x+y) - 3b(2x+y)$

$(4a - 3b)(2x+y)$

Factorise each of the following by grouping terms.

16. $6x^2 - 3y(3x - 2a) - 4ax$

17. $3ax^2 - 3ay^2 - 4bx^2 + 4by^2$

$3a(x^2 - y^2) \mp 4b(\pm x^2 \mp y^2)$

$(3a - 4b)(x^2 - y^2)$ ✓

$(3a - 4b)(x+y)(x-y)$

Using the difference of two squares, factorise the following:

20. $9x^2 - y^2$

$(3x+y)(3x-y)$

21. $16x^2 - 25y^2$

$(4x+5y)(4x-5y)$

Using the difference of two squares, factorise the following:

28. $45 - 5x^2$

$5(9-x^2)$

$5(3+x)(3-x)$

29. $45a^2 - 20$

$5(9a^2-4)$

$=5(3a+2)(3a-2)$

Using the difference of two squares, factorise the following:

32. $a^4 - b^4$

Difference of 2 squares

$(a^2+b^2)(a^2-b^2)$

$(a^2+b^2)(a+b)(a-b)$

Factorise each of the following quadratic expressions:

39. $3x^2 - 17x + 20$ **40.** $7x^2 - 18x + 8$

Q.39

+20
1×20
2×10
4×5 ✓

$3x^2-17x+20$

$-5x$

$(3x-5)(x-4)$ ✓

$-12x$

Q40

8
8×1
4×2 ✓

$7x^2-18x+8$

$-4x$

$(7x-4)(x-2)$ ✓

$-14x$

51. Using the quadratic formula, factorise each of the following: (ii) $x^2 + 2\sqrt{5}x - 15$

$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a=1$, $b=2\sqrt{5}$, $c=-15$

$X = \frac{-2\sqrt{5} \pm \sqrt{(2\sqrt{5})^2 - 4(1)(-15)}}{2(1)}$

$= \frac{-2\sqrt{5} \pm \sqrt{80}}{2} = \frac{-2\sqrt{5} \pm 4\sqrt{5}}{2}$

$= \sqrt{5} \pm 2\sqrt{5}$

$\Rightarrow x = 3\sqrt{5}$ or $x = -\sqrt{5}$

FACTOR $(x - 3\sqrt{5})(x + \sqrt{5})$