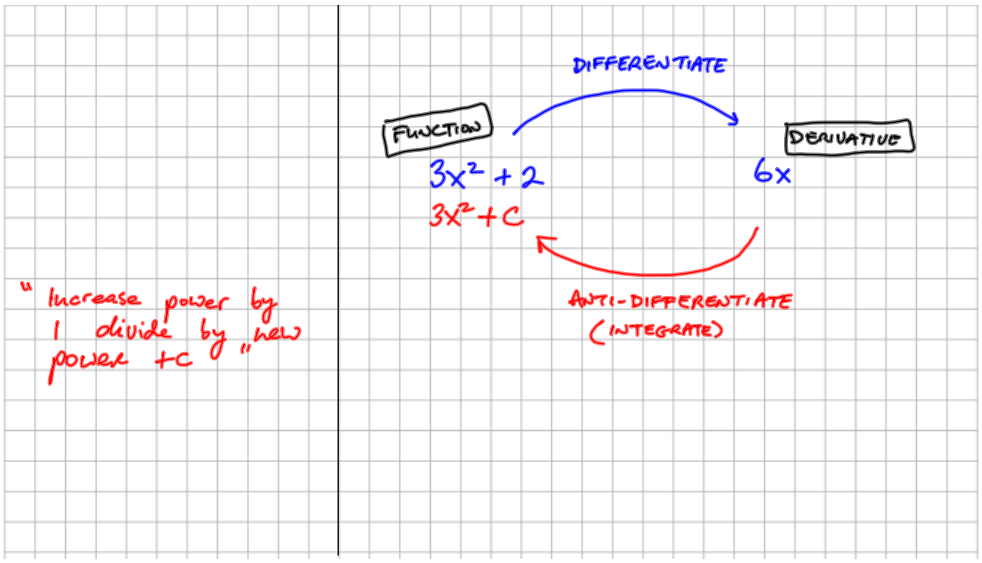
chapter 4 Integration

Section 4.1 Introduction to Integration



6. Find (i) $\int \frac{x^4 - 3x^3 + 4x}{x}\,dx$ (ii) $\int \frac{3x^3 - x^2 + 6}{x^2}\,dx$ (iii) $\int \frac{x^2 - 2x + 6}{\sqrt{x}}\,dx$

Simplify (i)

"Increase the power by 1 divide by new power + c"

$= \int (x^3 - 3x^2 + 4)\,dx$

$= \frac{x^4}{4} - \frac{3x^3}{3} + 4x + c$

$= \frac{x^4}{4} - x^3 + 4x + c$

Simplify (ii)

$= \int (3x - 1 + 6x^{-2})\,dx$

$= \frac{3x^2}{2} - x + \frac{6x^{-1}}{-1} + c$

$= \frac{3x^2}{2} - x - 6x^{-1} + c$

Simplify (iii)

$\sqrt{x} = x^{\frac{1}{2}}$

$= \int (x^{3/2} - 2x^{1/2} + 6x^{-1/2})\,dx$

$= \frac{x^{5/2}}{(5/2)} - \frac{2x^{3/2}}{(3/2)} + \frac{6x^{1/2}}{(1/2)} + c$

$= \frac{2}{5}x^{5/2} - \frac{4}{3}x^{3/2} + 12x^{1/2} + c$

7. A curve with equation $y = f(x)$ contains the point $(-1, 4)$. If $f'(x) = 2x$, find the equation of the curve.

Integrate because:

$\int \underbrace{f'(x)}_{\text{derivative}}\,dx = \underbrace{f(x)}_{\text{function}}$

$f(x) = \int 2x\,dx = \frac{2x^2}{2} + c$

$= x^2 + c$

function contains $(-1, 4)$

ie .. $f(-1) = 4$

$f(-1) = 4$

$(-1)^2 + c = 4$

$1 + c = 4 \Rightarrow c = 3$

If $c = 3$ $\Rightarrow$ $f(x) = x^2 + 3$

Section 4.2 Integrating exponential and trigonometric functions

INTEGRATE

$\cos ax \rightarrow \frac{1}{a}\sin ax$

$\sin ax \rightarrow -\frac{1}{a}\cos ax$

$e^{ax} \rightarrow \frac{1}{a}e^{ax}$

$e^x \rightarrow e^x$

Exercise 4.2

1. Find the following integrals:

(i) $\int e^{2x}\,dx$ (ii) $\int 3e^{x}\,dx$ (iii) $\int 2e^{4x}\,dx$ (iv) $\int e^{-3x}\,dx$

Integrate

$e^{x} \rightarrow e^{x}$

$e^{ax} \rightarrow \frac{1}{a}e^{ax}$

(i) $\int e^{2x}\,dx = \frac{1}{2}e^{2x} + c$

(ii) $\int 3e^{x}\,dx = 3e^{x} + c$

(iii) $\int 2e^{4x}\,dx = 2\left(\frac{1}{4}\right)e^{4x} + c$

$= \frac{1}{2}e^{4x} + c$

(iv) $\int e^{-3x}\,dx = \frac{1}{-3}e^{-3x} + c$

$= -\frac{1}{3}e^{-3x} + c$

Syllabus

- recognise integration as the reverse process of differentiation
- use integration to find the average value of a function over an interval

- integrate sums, differences and constant multiples of functions of the form
  - x^a, where $a \in \mathbf{Q}$
  - a^x, where $a \in \mathbf{R}$
  - Sin ax, where $a \in \mathbf{R}$
  - Cos ax, where $a \in \mathbf{R}$
- determine areas of plane regions bounded by polynomial and exponential curves

Maths Tables

Integration

Constants of integration omitted.

$f(x)$	$\int f(x)dx$
$x^n, (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln\lvert x\rvert$
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax}$
$a^x \ (a>0)$	$\frac{a^x}{\ln a}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\tan x$	$\ln\lvert\sec x\rvert$
$\frac{1}{\sqrt{a^2-x^2}} \ (a>0)$	$\sin^{-1}\frac{x}{a}$
$\frac{1}{x^2+a^2} \ (a>0)$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$

learn

$\cos ax \rightarrow \frac{1}{a}\sin ax$

$\sin ax \rightarrow -\frac{1}{a}\cos ax$