12. A pump is inflating a spherical balloon.
The radius of the balloon is increasing at a rate of 1 cm/sec when the length of the radius is 2 metres.

(i) Find the rate at which the pump is working (i.e. the rate of change of the volume).
(ii) If the air continues to be pumped at this rate, find the rate of change of the radius when it is 5 metres in length.
(iii) At what rate is the surface area of the balloon increasing when $r = 5$ m?

$\frac{dR}{dt} = 1$ cm/s ①

when $R = 2m = 200$ cm

$\frac{dV}{dt} = ?$ use:

$V = \frac{4}{3}\pi R^3$

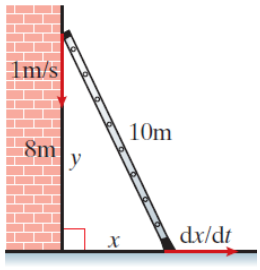
$\frac{dV}{dt} = \frac{dR}{dt} \cdot \frac{dV}{dR}$ ① ②

$\frac{dV}{dR} = 4\pi R^2$ ②

$\frac{dV}{dt} = (1)(4\pi R^2) = 4\pi R^2$

when $R = 200$ cm $\frac{dV}{dt} = 4\pi(200)^2 = 160{,}000\pi$ cm³/s

13. A 10m ladder leans against a vertical wall.
The top of the ladder is at a height of 8m up the wall and is slipping down the wall at a rate of 1m/sec.
At what rate is the foot of the ladder sliding along the ground when the top of the ladder is 8m up the wall?



$\frac{dx}{dt} = ?$

Given $\frac{dy}{dt} = 1$ m/s

key: $\frac{dx}{dt} = \frac{dy}{dt} \cdot \frac{dx}{dy}$

$\frac{dx}{dy} = ?$ Pythagoras $10^2 = x^2 + y^2$

$x^2 = 100 - y^2 \Rightarrow x = (100 - y^2)^{\frac{1}{2}}$

differentiate $\frac{dx}{dy} = \frac{1}{2}(100 - y^2)^{-\frac{1}{2}}(-2y)$

$= -y(100 - y^2)^{-\frac{1}{2}}$

$\Rightarrow \frac{dx}{dt} = (1)(-y(100 - y^2)^{-\frac{1}{2}})$

$y = 8$ $\frac{dx}{dt} = (-8)((100 - (-8)^2)^{-\frac{1}{2}}) = -\frac{4}{3}$ m/s