

Section 3.6 Related rates of change





In many situations, we are given one rate of change and asked to find another rate of change. In these situations, the *Chain Rule* is particularly useful when finding the required rate of change.

The Chain Rule states that $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

This can be extended to $\frac{dy}{dx} = \frac{dy}{d\bullet} \cdot \frac{d\bullet}{dx}$, where \bullet stands for any variable.

For example, $\frac{dA}{dr} = \frac{dA}{dt} \cdot \frac{dt}{dr}$

If we are given that $\frac{dA}{dr} = 8$, then $\frac{dr}{dA} = \frac{1}{8}$.

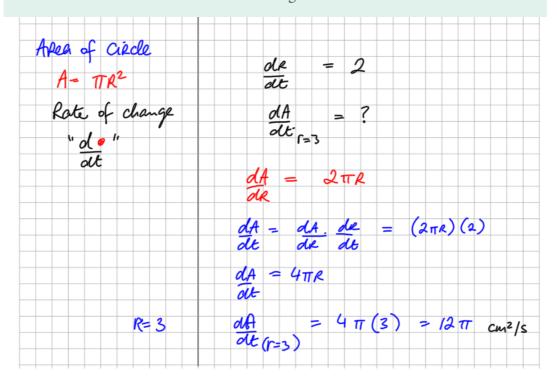
Similarly, if $\frac{dV}{dt} = 6\pi$, then $\frac{dt}{dV} = \frac{1}{6\pi}$.

$$\frac{\mathrm{d}t}{\mathrm{d}r} = \frac{1}{\frac{\mathrm{d}r}{\mathrm{d}t}}$$

The method of solving problems involving 'related rates' is illustrated in the following examples.

Example 1

The radius of a circle is increasing at the rate of 2 cm/sec. Find the rate at which the area is increasing when the radius is 3 cm.



When solving problems with related rates of change, these steps should prove useful.

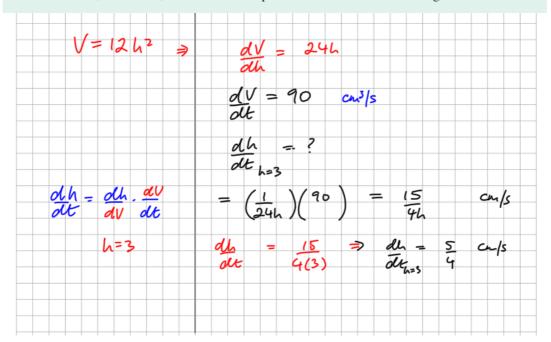
- 1. Write down the rate you require, e.g. $\frac{dA}{dt}$.
- 2. Write down the rate you are given, e.g. $\frac{dr}{dt}$.
- 3. Use the Chain Rule to link these rates: $\frac{dA}{dt} = \frac{dA}{?} \cdot \frac{?}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$
- **4.** Find an equation connecting the variables of the missing rate, i.e. $A = \pi r^2$.

Example 2

The volume, $V \text{ cm}^3$, of water in a container is given by the expression $V = 12h^2$, where h cm is the depth of the water.

Water is flowing into the container at a steady rate of 90 cm³/sec.

Find the rate, in cm/sec, at which the depth of the water is increasing when h = 3.



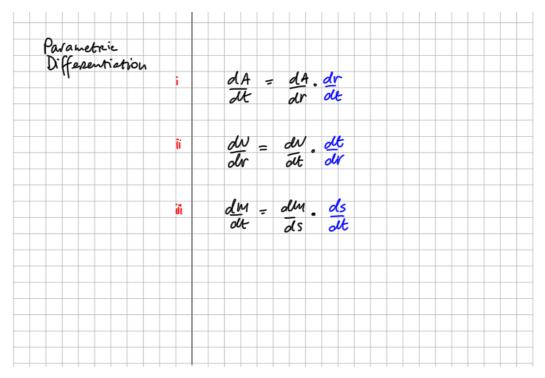
Exercise 3.6

1. In each of the following, fill in the missing rate:

(i)
$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}r} \cdot -$$

(ii)
$$\frac{dV}{dr} = \frac{dV}{dt} \cdot \dots$$

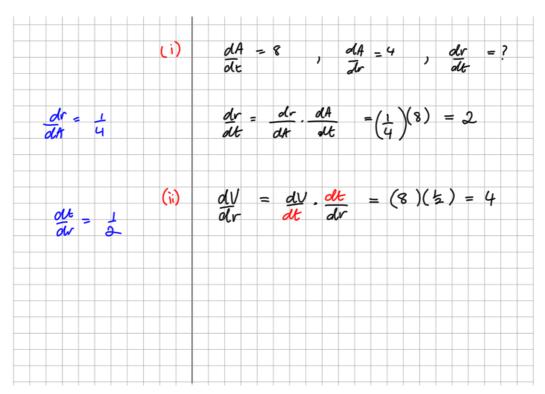
(i)
$$\frac{dA}{dt} = \frac{dA}{dr} \cdot$$
 (ii) $\frac{dV}{dr} = \frac{dV}{dt} \cdot$ (iii) $\frac{dM}{dt} = \frac{dM}{ds} \cdot$



2. In each of the following, find the indicated rate.

(i)
$$\frac{dA}{dt} = 8$$
, $\frac{dA}{dr} = 4$, $\frac{dr}{dt} = 4$

(i)
$$\frac{dA}{dt} = 8$$
, $\frac{dA}{dr} = 4$, $\frac{dr}{dt} = ?$ (ii) $\frac{dV}{dt} = 8$, $\frac{dr}{dt} = 2$, $\frac{dV}{dr} = ?$



3. If $\frac{dy}{dx} = 10$ and $\frac{dx}{dt} = 2$, find $\frac{dy}{dt}$.

