Chapter 3 Applications of Differential Calculus

Section 3.4 Maximum and minimum problems

Key idea: At max. or min. the derivative of the function $= 0$.

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Example 1

A factory produces n items per hour.

The overhead costs, $C(n) = €\left(400 - 16n + \frac{n^2}{4}\right)$.

How many items should be produced to keep the overhead costs to a minimum?

At min $C'(n) = 0$

$$C(n) = 400 - 16n + \frac{n^2}{4}$$

$$C'(n) = -16 + \frac{1}{2}n$$

$$-16 + \frac{1}{2}n = 0$$

$$\frac{1}{2}n = 16$$

$$n = 32 \quad \text{items per hour.}$$

Example 2

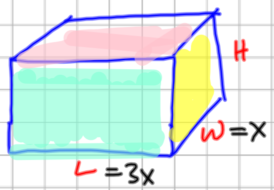
A closed rectangular box is made of thin metal. The length of the box is three times its width. The volume of the box is 36 cm³ and its width is x cm.

Show that the surface area is $\left(6x^2 + \frac{96}{x}\right)$ cm².

Find the dimensions of the box with least surface area.

$H = ?$

$\boxed{V = LWH}$



$V = 36\ \text{cm}^3$

$36 = (3x)(x)H \Rightarrow H = \frac{36}{3x^2}$

$\Rightarrow \boxed{H = \frac{12}{x^2}}$

$SA = 2[LW + WH + LH]$

$SA = 2\left[(3x)(x) + (3\cancel{x})\left(\frac{12}{\cancel{x^2}}\right) + (\cancel{x})\left(\frac{12}{\cancel{x^2}}\right)\right]$

$= 2\left[3x^2 + \frac{36}{x} + \frac{12}{x}\right]$

$= 2\left[3x^2 + \frac{48}{x}\right] = 6x^2 + \frac{96}{x}$ QED

$A = 6x^2 + \frac{96}{x} = 6x^2 + 96x^{-1}$

at min

$\frac{dA}{dx} = 0$

$\frac{dA}{dx} = 12x - 96x^{-2}$

at min $12x - 96x^{-2} = 0$

$12x - \frac{96}{x^2} = 0$

× LCD i.e. x^2

$12x^3 - 96 = 0$

$12x^3 = 96$

$x^3 = \frac{96}{12}$

$x^3 = 8$

$x = 2$