7. A closed rectangular box with a square base of side x cm and height h cm has a total surface area of 54 cm^2.

(i) Express h in terms of x.

(ii) Express the volume of the box in terms of x.

(iii) For what value of x is the volume at a maximum? Hence, find this volume.

(i) $h = ?$

$TSA = 54 \text{ cm}^2$

$SA = 2[LB + HB + HL]$

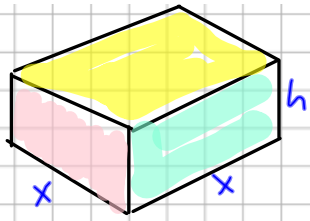
(ii) $V = ?$ (in terms of x)

$V = LBH$

(iii) When V_{max}, $x = ?$

$\frac{dV}{dx} = 0$ at max

Hence $V = ?$



$2[x^2 + xh + xh] = 54$

$x^2 + 2xh = 27$

$2xh = 27 - x^2$

$h = \frac{27 - x^2}{2x}$

$V = (x)(x)(h) \Rightarrow V = x^2\left(\frac{27 - x^2}{2x}\right)$

$\Rightarrow V = x\left(\frac{27 - x^2}{2}\right) \Rightarrow V = \frac{27}{2}x - \frac{1}{2}x^3$

$\frac{dV}{dx} = \frac{27}{2} - \frac{3}{2}x^2 = 0$

$3x^2 = 27 \Rightarrow x^2 = 9 \Rightarrow x = 3 \text{ cm}$

$V = \frac{27}{2}(3) - \frac{1}{2}(3)^3 \Rightarrow V = 27 \text{ cm}^3$