Example 2

If $y = \overset{u}{e^{2x}} . \overset{v}{\cos 2x}$, find the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{8}$.

Product rule

$$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$u = e^{2x}$, $\frac{du}{dx} = 2e^{2x}$

$v = \cos 2x$, $\frac{dv}{dx} = -2\sin 2x$

$$\frac{dy}{dx} = (e^{2x})(-2\sin 2x) + (\cos 2x)(2e^{2x})$$

$$= 2e^{2x}(\cos 2x - \sin 2x)$$

$x = \pi/8$ PUT CALCULATOR IN RADS

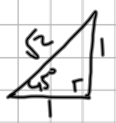
$$\left.\frac{dy}{dx}\right|_{(x=\pi/8)} = 2e^{2(\frac{\pi}{8})}\left(\cos 2\left(\frac{\pi}{8}\right) - \sin 2\left(\frac{\pi}{8}\right)\right)$$

$$= 2e^{(\pi/4)}\left[\cos \pi/4 - \sin \pi/4\right]$$

$$= 2e^{(\pi/4)} . \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right]$$

$$= 0$$

$\frac{\pi}{4} = 45°$



4. Differentiate each of these:

(i) $y = e^{2x}(1 + e^x)$ PRODUCT

(ii) $t = \frac{e^{2x}}{x}$ QUOTIENT

(iii) $\overset{u . v}{x^2 e^{\cos x}}$ PRODUCT, CHAIN

(iii)

Product rule

$$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$u = x^2$

$\frac{du}{dx} = 2x$

$v = e^{\cos x}$

$$\frac{dv}{dx} = \underbrace{e^{\cos x}(-\sin x)}_{\text{CHAIN RULE}}$$

$$= -\sin x . e^{\cos x}$$

$$\frac{dy}{dx} = (x^2)(-\sin x e^{\cos x}) + (e^{\cos x})(2x)$$

$$= -x^2 \sin x e^{\cos x} + 2x e^{\cos x}$$

4. Differentiate each of these:

(i) $y = e^{2x}(1 + e^x)$ (ii) $t = \dfrac{e^{2x}}{x}$ (iii) $x^2 e^{\cos x}$

Quotient rule

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$u = e^{2x}$ $\quad \frac{du}{dx} = 2e^{2x}$

$V = x$ $\quad \frac{dv}{dx} = 1$

$$t = \frac{e^{2x}}{x}$$

$$\frac{dt}{dx} = \frac{(x)(2e^{2x}) - (e^{2x})(1)}{x^2}$$

$$= \frac{e^{2x}(2x-1)}{x^2}$$