4. Differentiate each of these:

(i) $y = e^{2x}(1 + e^x)$ PRODUCT

(ii) $t = \frac{e^{2x}}{x}$ QUOTIENT

(iii) $x^2 e^{\cos x}$ u.v PRODUCT, CHAIN

(iii)

Product rule

$$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$u = x^2$

$\frac{du}{dx} = 2x$

$v = e^{\cos x}$

$\frac{dv}{dx} = \underbrace{e^{\cos x}(-\sin x)}_{\text{CHAIN RULE}}$

$= -\sin x . e^{\cos x}$

$$\frac{dy}{dx} = (x^2)(-\sin x e^{\cos x}) + (e^{\cos x})(2x)$$

$$= -x^2 \sin x e^{\cos x} + 2x e^{\cos x}$$

10. If $y = e^{mx}, m \in R$, find $\frac{d^2y}{dx^2}$.

Hence, find m if $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0$.

$f(x) \rightarrow f'(x)$

$e^{ax} \quad ae^{ax}$

$y = e^{mx}$

$\frac{dy}{dx} = me^{mx}$

$\frac{d^2y}{dx} = m^2 e^{mx}$

Hence

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0$

$m^2e^{mx} - 3(me^{mx}) - 4e^{mx} = 0$

$m^2e^{mx} - 3me^{mx} - 4e^{mx} = 0$

$(m^2 - 7m)e^{mx} = 0$

$\Rightarrow m^2 - 7m = 0$

$m(m-7) = 0$

$\Rightarrow m = 0 \quad \text{OR} \quad m = 7$

11. Let $f(x) = \dfrac{e^x + e^{-x}}{2}$

Show that $f''(x) = f(x)$, where $f''(x)$ is the second derivative of $f(x)$.

Write as Separate fractions

$$f(x) = \frac{e^x + e^{-x}}{2} = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$$

$f(x) \rightarrow f'(x)$

$e^{ax} \rightarrow ae^{ax}$

$$f'(x) = \frac{1}{2}[e^x] + \frac{1}{2}[-1e^{-x}]$$
$$= \frac{1}{2}e^x - \frac{1}{2}e^{-x}$$

Second derivative

$$f''(x) = \frac{1}{2}e^x - \frac{1}{2}[-1e^{-x}]$$
$$= \frac{1}{2}e^x + \frac{1}{2}e^{-x}$$
$$= \frac{e^x + e^{-x}}{2}$$

QED

12. Find the equation of the tangent to the curve $y = 3e^x - \sin x + 5$ at the point where $x = 0$.

The slope when $x=0$ is $f'(0)$

$f(x) \rightarrow f'(x)$

$e^{ax} \rightarrow ae^{ax}$

$\sin x \rightarrow \cos x$

$$y = 3e^x - \sin x + 5$$
$$\frac{dy}{dx} = 3e^x - \cos x$$

Sub in $x=0$

$$\frac{dy}{dx}_{(x=0)} = 3e^0 - \cos(0) = 2$$

Slope

$\boxed{m = 2}$ when $x = 0$

$x=0$, $y=?$

$$y = 3e^0 - \sin(0) + 5 \Rightarrow \boxed{y = 8}$$

Line/Tangent

$y - y_1 = m(x - x_1)$

$$y - 8 = 2(x - 0)$$
$$y - 8 = 2x$$
$$y = 2x + 8$$