Differential Calculus

chapter 2

Section 2.8 Differentiation of exponential functions

PROJECT MATHS Text & Tests 7

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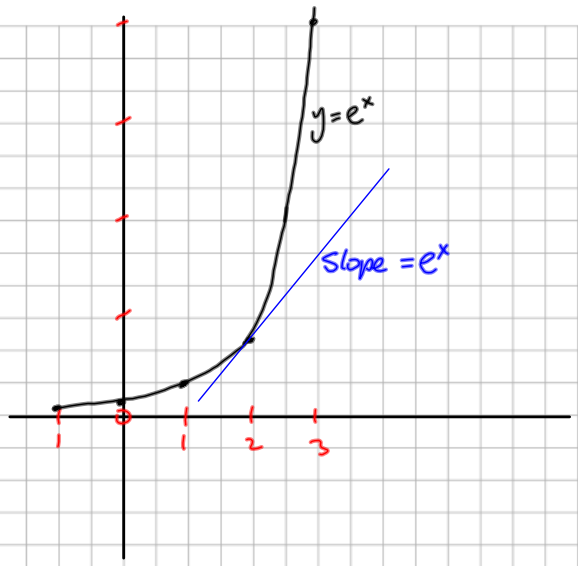
x	e^x
-1	0.4
0	1
1	2.7
2	7.4
3	20.1



In the function $y = e^x$

its slope at any point is equal to its y-value!

Example 1

If $y = e^x$, then $\frac{dy}{dx} = e^x$.

Find $\frac{dy}{dx}$ for each of the following:

(i) $y = 5e^{x^2}$ (ii) $y = e^{\cos x}$ (iii) $y = (e^x + 1)^4$

$f(x) \to f'(x)$
$e^x \to e^x$

CHAIN RULE

(i) $y = 5e^{x^2}$

$\frac{dy}{dx} = \underbrace{5e^{x^2}} \cdot \underbrace{(2x)} = 10x\, e^{x^2}$

(ii) $y = e^{\cos x}$

$y = \underbrace{e^{\cos x}} \cdot \underbrace{(-\sin x)} = -\sin x \cdot e^{\cos x}$

CHAIN RULE
$f(x) \to f'(x)$
$\cos x \to -\sin x$

(iii) $y = (e^x + 1)^4$

$\frac{dy}{dx} = \underbrace{4(e^x+1)^3} \cdot \underbrace{e^x} = 4e^x(e^x+1)^3$

CHAIN RULE

1. Find $\frac{dy}{dx}$ for each of the following:

(iv) $y = e^{2x+4}$ (v) $y = e^{x^2+3x}$ (vi) $y = e^{\sin x}$

$f(x) \to f'(x)$
$e^x \to e^x$

CHAIN RULE IN EACH QUESTION

(iv) $y = e^{2x+4}$

$\frac{dy}{dx} = \underbrace{e^{2x+4}} \cdot \underbrace{(2)} = 2e^{2x+4}$

(v) $y = e^{x^2+3x}$

$\frac{dy}{dx} = \underbrace{e^{x^2+3x}} \cdot \underbrace{(2x+3)}$

$= (2x+3)e^{x^2+3x}$

(vi) $y = e^{\sin x}$

$\frac{dy}{dx} = \underbrace{e^{\sin x}} \cdot \underbrace{(\cos x)}$

$= \cos x \cdot e^{\sin x}$

2. Differentiate each of these:

(i) $y = e^{\frac{x}{2}}$ (ii) $y = e^{\sin^2 x}$ (iii) $y = xe^{2x}$

(ii)

3 layer chain Rule

$y = e^{\sin^2 x} = e^{(\sin x)^2}$

$\frac{dy}{dx} = \underbrace{e^{\sin^2 x}} \cdot \underbrace{2(\sin x)^1} \cdot \underbrace{(\cos x)}$

$= 2\sin x \cos x \, e^{\sin^2 x}$

(iii)

Product rule

$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$u = x$, $\frac{du}{dx} = 1$

$v = e^{2x}$, $\frac{dv}{dx} = 2e^{2x}$

$f(x) \to f'(x)$

$e^{ax} \to ae^{ax}$

$\frac{dy}{dx} = (x)(2e^{2x}) + (e^{2x})(1)$

$= 2xe^{2x} + e^{2x}$

OR $= (2x+1)e^{2x}$

4. Differentiate each of these:

(i) $y = e^{2x}(1 + e^x)$ (ii) $t = \frac{e^{2x}}{x}$ (iii) $x^2 e^{\cos x}$

PRODUCT QUOTIENT PRODUCT, CHAIN

u.v

(iii)

Product rule

$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$u = x^2$

$\frac{du}{dx} = 2x$

$v = e^{\cos x}$

$\frac{dv}{dx} = \underbrace{e^{\cos x}(-\sin x)}_{\text{CHAIN RULE}}$

$= -\sin x \cdot e^{\cos x}$

$\frac{dy}{dx} = (x^2)(-\sin x e^{\cos x}) + (e^{\cos x})(2x)$

$= -x^2 \sin x \, e^{\cos x} + 2x \, e^{\cos x}$

10. If $y = e^{mx}, m \in R$, find $\frac{d^2y}{dx^2}$.

Hence, find m if $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0$.

$f(x) \rightarrow f'(x)$

$e^{ax} \qquad ae^{ax}$

$y = e^{mx}$

$\frac{dy}{dx} = me^{mx}$

$\frac{d^2y}{dx} = m^2e^{mx}$

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0$

Hence

$m^2e^{mx} - 3(me^{mx}) - 4e^{mx} = 0$

$m^2e^{mx} - 3me^{mx} - 4e^{mx} = 0$

$(m^2 - 7m)e^{mx} = 0$

$\Rightarrow m^2 - 7m = 0$

$m(m-7) = 0$

$\Rightarrow m = 0$ OR $m = 7$