



Section 2.7 Differentiation of inverse trigonometric functions



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Standard derivatives
of inverse functions

$$f(x) = \sin^{-1} \left(\frac{x}{a} \right) \Rightarrow f'(x) = \frac{1}{\sqrt{a^2 - x^2}}$$

$$f(x) = \tan^{-1} \left(\frac{x}{a} \right) \Rightarrow f'(x) = \frac{a}{a^2 + x^2}$$

Example 1

If $y = \sin^{-1} \frac{5x}{3}$, find $\frac{dy}{dx}$.

$$a = \frac{3}{5}$$

$$y = \sin^{-1} \left(\frac{5x}{3} \right) = \sin^{-1} \left(\frac{x}{\frac{3}{5}} \right)$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sqrt{\left(\frac{3}{5}\right)^2 - x^2}} \\ &= \frac{1}{\sqrt{\frac{9}{25} - x^2}} \end{aligned}$$

Standard derivatives
of inverse functions

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Example 2

If $y = \tan^{-1} (2x+1)$, find $\frac{dy}{dx}$.

a

CHAIN RULE

$$\frac{dy}{dx} = \left(\frac{1}{1^2 + (2x+1)^2} \right) \cdot (2)$$

$$= \frac{2}{1 + 4x^2 + 4x + 1}$$

$$= \frac{2}{4x^2 + 4x + 2} = \cancel{\frac{2}{2(2x^2 + 2x + 1)}}$$

$$= \frac{1}{2x^2 + 2x + 1}$$