6. If $f(x) = \cos x . \tan x$, show that $f'(x) = \cos x$.

$u \quad v$

Product rule

$$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$v = \tan x$

$\frac{dv}{dx} = \sec^2 x$

$\sec x = \frac{1}{\cos x}$

$\tan x = \frac{\sin x}{\cos x}$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow 1 - \sin^2 x = \cos^2 x$

$f'(x) = (\cos x)(\sec^2 x) + (\tan x)(-\sin x)$ ✓

$= \cancel{\cos x}\left(\frac{1}{\cos^{\cancel{2}} x}\right) + \frac{\sin x}{\cos x}(-\sin x)$

$= \frac{1 - \sin^2 x}{\cos x}$

$= \frac{\cos^{\cancel{2}} x}{\cancel{\cos x}}$

$= \cos x$

OR $f(x) = \cos x . \tan x = \cancel{\cos x}\,\frac{\sin x}{\cancel{\cos x}}$

$f'(x) = \cos x$

8. Given that $\tan x = \frac{\sin x}{\cos x}$, use the *Quotient Rule* to show that $\frac{d}{dx}(\tan x) = \sec^2 x$.

$f'(x) = ?$

Quotient rule

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$v = \cos x$

$\frac{dv}{dx} = -\sin x$

$\boxed{\cos^2 x + \sin^2 x = 1}$

$\frac{1}{\cos x} = \sec x$

$f(x) = \tan x = \frac{\sin x}{\cos x} \quad \begin{matrix} u \\ v \end{matrix}$

$f'(x) = \frac{(\cos x)(\cos x) + (\sin x)(+\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$

$= \left(\frac{1}{\cos x}\right)^2$

$= (\sec x)^2$

$= \sec^2 x$