Differential Calculus

Chapter 2

Section 2.6 The derivatives of trigonometric functions

PROJECT MATHS Text & Tests 7

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\tan x) = \sec^2 x$$

Example 1

Differentiate each of the following with respect to x:

(i) $y = 3\sin x + 2\cos x$ (ii) $y = x^2 \sin x$

(i) $f(x) \to f'(x)$

$\sin x \to \cos x$

$\cos x \to -\sin x$

$y = 3\sin x + 2\cos x$

$\frac{dy}{dx} = 3\cos x - 2\sin x$

(ii) PRODUCT RULE

Product rule

$$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$y = x^2 \cdot \sin x$

$u \cdot v$

$u = x^2 \quad \frac{du}{dx} = 2x$

$v = \sin x \quad \frac{dv}{dx} = \cos x$

$\frac{dy}{dx} = (x^2)(\cos x) + (\sin x)(2x)$

$= x^2\cos x + 2x\sin x$

$\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\cos x) = -\sin x$ $\frac{d}{dx}(\tan x) = \sec^2 x$

Example 2

Find the derivative of each of the following:

(i) $\cos(7x - 3)$ (ii) $\tan^2 3x$ (iii) $\sin^3(x^2 + 2)$

CHAIN RULE

(iii) $y = \sin^3(x^2+2) = [\sin(x^2+2)]^3$

$\frac{dy}{dx} = 3[\sin(x^2+2)]^2 \cdot \cos(x^2+2) \cdot 2x$

DIFF OUTSIDE × DIFF MIDDLE × DIFF INSIDE

$= 6x \sin^2(x^2+2)\cos(x^2+2)$

2. Find $\frac{dy}{dx}$ for each of these: CHAIN RULE

(iv) $y = \sin^3(4x)$ (v) $y = \cos^2(2x + 1)$ (vi) $y = \tan^3(4x + 3)$

(iv) $y = \sin^3(4x) = [\sin(4x)]^3$

$\frac{dy}{dx} = 3[\sin(4x)]^2 \cdot \cos(4x) \cdot 4$

$= 12\sin^2(4x)\cos(4x)$

(v) $y = \cos^2(2x+1) = [\cos(2x+1)]^2$

$\frac{dy}{dx} = 2[\cos(2x+1)]^1 \cdot [-\sin(2x+1)] \cdot 2$

$= -4\cos(2x+1)\sin(2x+1)$

(vi) $y = \tan^3(4x+3) = [\tan(4x+3)]^3$

$\frac{dy}{dx} = 3[\tan(4x+3)]^2 \cdot \sec^2(4x+3) \cdot 4$

$= 12\tan^2(4x+3)\sec^2(4x+3)$