Differential Calculus

chapter 2

Section 2.5 Second derivatives

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PROJECT MATHS

Text & Tests 7

For any function $y = f(x)$, the first derivative is $\frac{dy}{dx}$ or $f'(x)$.

If we differentiate the resulting function, we get the **second derivative**.

The second derivative is denoted by $\mathbf{\frac{d^2y}{dx^2}}$ or $\boldsymbol{f''(x)}$.

$\frac{d^2y}{dx^2}$ is pronounced 'dee two *y* dee *x* squared'.

We learned earlier that for any function $y = f(x)$, $\frac{dy}{dx}$ represents the slope of the tangent to the curve at any point on the curve. When dealing with the graphs of functions in the next chapter, we will see that $\frac{d^2y}{dx^2}$ gives the rate at which the slope is changing over a given interval.

Example 1

Given that $y = x + \frac{1}{x}$, find $\frac{d^2y}{dx^2}$.

Write in index notation	$y = x + x^{-1}$
FIRST DERIVATIVE	$\frac{dy}{dx} = 1 - x^{-2}$ OR $1 - \frac{1}{x^2}$
$x^{-2} = \frac{1}{x^2}$	
SECOND DERIVATIVE	$\frac{d^2y}{dx^2} = +2x^{-3}$ or $\frac{2}{x^3}$
$x^{-3} = \frac{1}{x^3}$	

Example 2

If $y = \frac{3}{x} + 4x$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$; hence, show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$.

Write in index notation	$y = 3x^{-1} + 4x$
FIRST DERIVATIVE	$\frac{dy}{dx} = -3x^{-2} + 4$
SECOND DERIVATIVE	$\frac{d^2y}{dx^2} = 6x^{-3}$
Hence ..	$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y$
	$= x^2(6x^{-3}) + x(-3x^{-2}+4) - (3x^{-1}+4x)$
	$= 6x^{-1} - 3x^{-1} + 4x - 3x^{-1} - 4x$
	$= \cancel{6x^{-1}} - \cancel{6x^{-1}} + \cancel{4x} - \cancel{4x}$
	$= 0$