Differential Calculus

chapter 2

Section 2.4 Product, Quotient and Chain Rules

PROJECT MATHS Text & Tests 7

Product Rule

If $y = uv$, where u and v are both functions of x, then
$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Example 1

If $y = (6x^2 + 2x)(3x - 2)$, find $\frac{dx}{dy}$.

($u = 6x^2 + 2x$, $v = 3x - 2$)

① By Product rule

$u = 6x^2 + 2x$

$\frac{du}{dx} = 12x + 2$

$V = 3x - 2$

$\frac{dv}{dx} = 3$

$\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

$\frac{dy}{dx} = (6x^2+2x)(3) + (3x-2)(12x+2)$

$= 18x^2 + 6x + 36x^2 + 6x - 24x - 4$

$= 54x^2 - 12x - 4$

② By expanding first

$y = (6x^2+2x)(3x-2) = 18x^3 - 12x^2 + 6x^2 - 4x$

$= 18x^3 - 6x^2 - 4x$

$\frac{dy}{dx} = 54x^2 - 12x - 4$

Quotient Rule

If $y = \frac{u}{v}$, where u and v are functions of x, then

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Example 2

If $f(x) = \frac{x^2+7}{3x-1}$ (u / v), find $f'(x)$.

$u = x^2+7$

$\frac{du}{dx} = 2x$

$v = 3x-1$

$\frac{dv}{dx} = 3$

careful!

$$\frac{dy}{dx} = \frac{(3x-1)(2x) - (x^2+7)(3)}{(3x-1)^2}$$ ← don't expand

$$= \frac{6x^2-2x-3x^2-21}{(3x-1)^2}$$

$$= \frac{3x^2-2x-21}{(3x-1)^2}$$

The Chain Rule

If y is a function of u, and u is a function of x,

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

"function of a function"

Example 3

Find $\frac{dy}{dx}$ if (i) $y = (2x^2-1)^3$ (ii) $y = \sqrt{3x^2-2}$.

CHAIN RULE
Diff. of outside
X Diff of inside

(i) $\frac{dy}{dx} = 3(2x^2-1)^2 \cdot 4x$

$= 12x(2x^2-1)^2$

(ii) $y = (3x^2-2)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(3x^2-2)^{-\frac{1}{2}} \cdot (6x)$

$= 3x(3x^2-2)^{-\frac{1}{2}}$

Example 4

Find $\frac{dy}{dx}$ if (i) $y = (x^2 - 3x)^4$ (ii) $y = \sqrt{x^2 - 6x}$.

CHAIN RULE:
DIFF. OF OUTSIDE
X DIFF. OF INSIDE

i $y = (x^2 - 3x)^4$

$\frac{dy}{dx} = \underbrace{4(x^2-3x)^3}_{\text{DIFF OF OUTSIDE}} \cdot \underbrace{(2x-3)}_{\text{DIFF OF INSIDE}}$

ii $y = \sqrt{x^2 - 6x} = (x^2 - 6x)^{\frac{1}{2}}$

$\frac{dy}{dx} = \underbrace{\frac{1}{2}(x^2-6x)^{-\frac{1}{2}}}_{\text{DIFF. OF OUTSIDE}} \cdot \underbrace{(2x-6)}_{\text{DIFF. OF INSIDE}}$