Differential Calculus

chapter **2**

Section 2.3 Differentiation by rule

$f(x)$	$f'(x)$
y	$\dfrac{dy}{dx}$
	multiply by power Reduce the power by 1
ax^n	$an\,x^{n-1}$
	derivative of a constant is zero
a	0

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Example 1

Find the derivative of $f(x) = 6x^3 - 3x^2 + 4x$.
Hence, find $f'(2)$ and interpret the result.

$f(x) \rightarrow f'(x)$
$ax^n \qquad anx^{n-1}$

$$f'(x) = 18x^2 - 6x + 4$$

$x = 2$

$$f'(2) = 18(2)^2 - 6(2) + 4$$
$$= 64$$

When $x = 2$
the slope of the tangent to the curve is 64

Example 2

Find $\dfrac{dy}{dx}$ for each of the following:

(i) $y = 3x^2 + \dfrac{2}{x} = 3x^2 + 2x^{-1}$

Write x terms in power notation

$\dfrac{dy}{dx} = 6x - 2x^{-2}$

(ii) $y = \sqrt{x} - \dfrac{4}{x^2} = x^{\frac{1}{2}} - 4x^{-2}$

$\dfrac{dy}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}} + 8x^{-3}$

Example 3

Find the slope of the tangent to the curve $y = 3x^2 + 4x - 5$ at the point $(1, 2)$.
Hence, find the equation of the tangent at this point.

$y = 3x^2 + 4x - 5$

differentiate

$\dfrac{dy}{dx} = 6x + 4$

at $(1,2)$ Slope = ?
$\Rightarrow x = 1$

$\dfrac{dy}{dx}\bigg|_{x=2} = 6(1) + 4 = 10$

Tangent is a line

$y - y_1 = m(x - x_1)$
$(1, 2) \qquad m = 10$
$x_1 \; y_1$

$y - 2 = 10(x - 1)$
$y - 2 = 10x - 10$
$10x - y - 8 = 0 \qquad$ *Tangent*

Example 4

Find the points on the curve $y = x^3 - 3x^2$ at which the slope of the tangent to the curve is 9.

derivative = Slope	$y = x^3 - 3x^2$
	$\frac{dy}{dx} = 3x^2 - 6x$ ✓
Slope = 9 ⟹	$3x^2 - 6x = 9$ ✓
÷3	$3x^2 - 6x - 9 = 0$
Solve	$x^2 - 2x - 3 = 0$
	$(x - 3)(x + 1) = 0$
	$x = 3 \ , \ x = -1$ ✓
y values ?	
on curve $y = x^3 - 3x^2$	$x = 3 \ , \ y = (3)^3 - 3(3)^2 = 0 \quad ⟹ \ \text{pt.} \ (3, 0)$
	$x = -1, \ y = (-1)^3 - 3(-1)^2 = -1 - 3 = -4 ⟹ \ \text{pt} \ (-1, -4)$ ✓

7. If $f(x) = \dfrac{1}{\sqrt{x}}$, find $f'(4)$ and express your answer as a fraction.

	$f(x) = \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-\frac{1}{2}}$
differentiate	$f'(x) = -\frac{1}{2} x^{-3/2}$
$f'(4) = ?$	$f'(4) = -\frac{1}{2} (4)^{-3/2}$
	$= -\frac{1}{16}$