Example 3

Find, from first principles, the slope of the tangent to the curve with equation $f(x) = x^2 + x + 5$ at the point where $x = 3$.

① $f(x) = x^2 + x + 5$

② $f(x+h) = (x+h)^2 + (x+h) + 5 = x^2 + 2xh + h^2 + x + h + 5$

③ $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2x\cancel{h} + h\cancel{^2} + \cancel{h}1}{\cancel{h}}$

④ $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = 2x + 0 + 1$

⑤ $\underbrace{f'(x)}_{\text{slope}} = 2x + 1$

$x = 3$, slope = ?

$f'(3) = 2(3) + 1 = 7$

Exercise 2.2

1. Differentiate each of these from first principles:

(i) $f(x) = 5x$ (ii) $f(x) = 3x - 4$ (iii) $f(x) = 6 - 4x$

① $f(x) = 5x$

② $f(x+h) = 5(x+h) = 5x + 5h$

③ $\dfrac{f(x+h) - f(x)}{h} = \dfrac{5\cancel{h}}{\cancel{h}}$

④ $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = 5$

⑤ $f'(x) = 5$

Exercise 2.2

1. Differentiate each of these from first principles:

(i) $f(x) = 5x$ (ii) $f(x) = 3x - 4$ (iii) $f(x) = 6 - 4x$

① $f(x) = 3x - 4$

② $f(x+h) = 3(x+h) - 4 = 3x + 3h - 4$

③ $\dfrac{f(x+h) - f(x)}{h} = \dfrac{+3\cancel{h}}{\cancel{h}}$

④ $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = 3$

⑤ $f'(x) = 3$

Exercise 2.2

1. Differentiate each of these from first principles:

(i) $f(x) = 5x$ (ii) $f(x) = 3x - 4$ (iii) $f(x) = 6 - 4x$

① $f(x) = 6 - 4x$

② $f(x+h) = 6 - 4(x+h) = 6 - 4x - 4h$

③ $\dfrac{f(x+h) - f(x)}{h} \quad \dfrac{-4\cancel{h}}{\cancel{h}}$

④ $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = -4$

⑤ $f'(x) = -4$

2. Find the derivatives of each of the following from first principles:

(i) $f(x) = x^2$ (ii) $f(x) = 2x^2 + 9x$ (iii) $f(x) = 3x^2 - 4x - 6$

① $f(x) =$	x^2
② $f(x+h) =$	$(x+h)^2 = x^2 + 2xh + h^2$
③ $\frac{f(x+h) - f(x)}{h}$	$\frac{2x\cancel{h} + h^{\cancel{2}}}{\cancel{h}}$
④ $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$	$2x + 0$
⑤ $f'(x) =$	$2x$

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① $f(x) =$	$2x^2 + 9x$
② $f(x+h) =$	$2(x+h)^2 + 9(x+h) = 2x^2 + 4xh + 2h^2 + 9x + 9h$
③ $\frac{f(x+h) - f(x)}{h}$	$\frac{4x\cancel{h} + 2h^{\cancel{2}} + 9\cancel{h}}{\cancel{h}}$
④ $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$	$4x + 2(0) + 9 = 4x + 9$
⑤ $f'(x) =$	$4x + 9$

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(i) $f(x) = x^2$ (ii) $f(x) = 2x^2 + 9x$ * (iii) $f(x) = 3x^2 - 4x - 6$

① $f(x) =$	$3x^2 - 4x - 6$
② $f(x+h) =$	$3(x+h)^2 - 4(x+h) - 6 = 3x^2 + 6xh + 3h^2 - 4x - 4h - 6$
③ $\dfrac{f(x+h) - f(x)}{h}$	$\dfrac{6x\cancel{h} + 3h^{\cancel{2}} - 4\cancel{h}}{\cancel{h}}$
④ $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} =$	$6x + 3(0) - 4$
⑤ $f'(x) =$	$6x - 4$