Functions

Chapter 1

Section 1.3 Types of functions

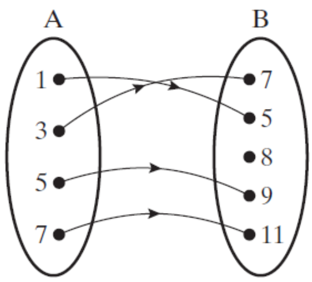
PROJECT MATHS
Text & Tests 7

1. Injective functions

A function *f*, from A to B, is said to be **injective** or **one-to-one** if every output in B has a unique input in A.

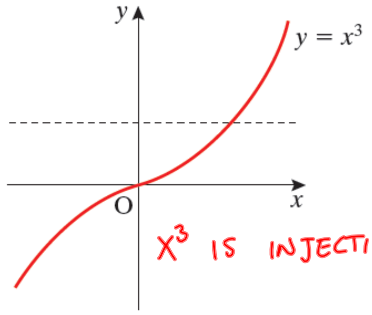
In a mapping diagram, no two inputs in A will have the same output in B; hence, the phrase 'one-to-one'.

Note: It is not necessary that every element in the codomain has a corresponding element in the domain.
In the diagram on the right, 8 has no corresponding input; however, the mapping diagram still represents an injective function.

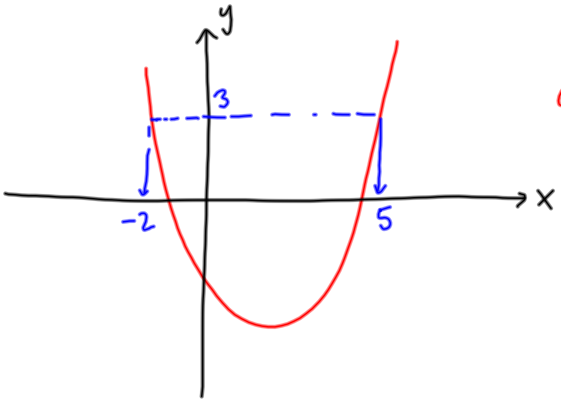


Horizontal line test for injective functions

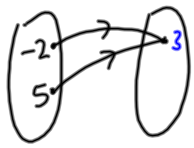
The diagram on the right shows the graph of $y = x^3$.
If $y = x^3$ is an injective function, then any line drawn parallel to the x-axis will intersect the curve **at most once**.
The diagram shows that $y = x^3$ is an injective function.



X^3 IS INJECTIVE



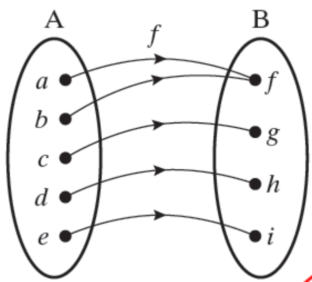
QUADRATIC IS NOT INJECTIVE



2. Surjective (many-to-one) functions

A function f is **surjective** if **every** element of the codomain is the image of **at least one** element of the domain.

A surjective function may also be described as a mapping of A **onto** B.
No element of the codomain is left unused.
Hence, the range and the codomain are equal.

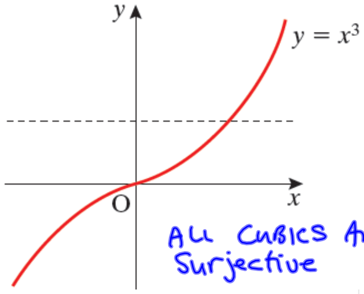


surjective ✓
injective X

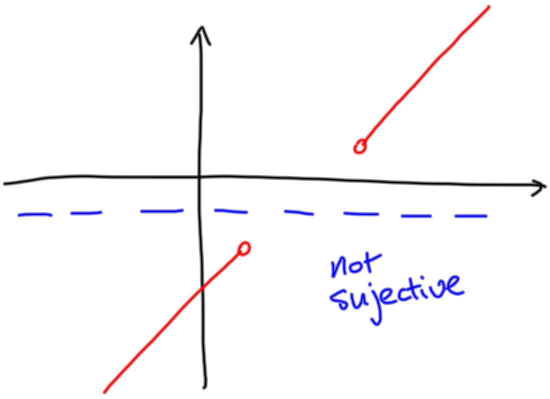
* For any function f, if the range and the codomain are equal, then the function is surjective.

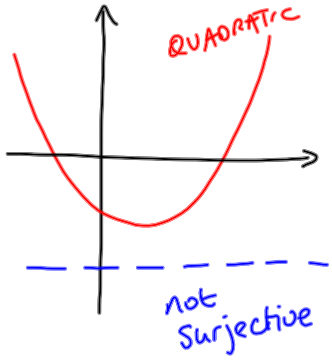
Horizontal line test for surjective functions

Consider the function $f: R \to R$.
If f is surjective, then every horizontal line drawn must intersect the graph **at least once**.
The function $y = x^3$, where $x \in R$, is surjective as every horizontal line intersects the graph at least once.



ALL CUBICS ARE Surjective



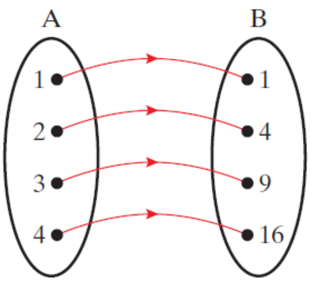


3. Bijective functions

A function f is **bijective** if for every element y in B, there is exactly one element x in A such that $f(x) = y$.

Therefore, a bijective function is both injective and surjective.

From the diagram, it can be seen that there is an exact one-to-one match between the elements in A and B.
This function is bijective.

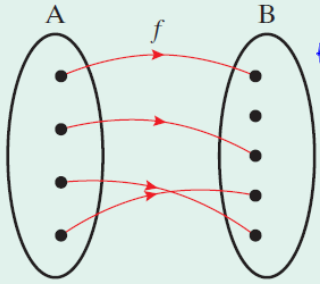


Surjective ✓
injective ✓
⇒ bijective

Example 1

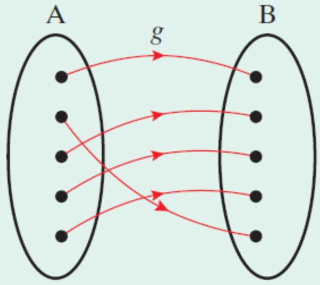
State whether the following maps A → B are
(i) functions (ii) injective (iii) surjective (iv) bijective.
Give reasons for your answers.

(a)



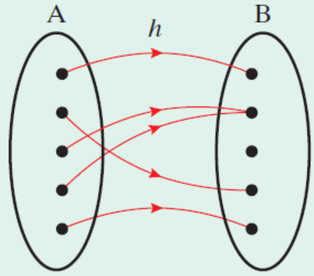
function ✓
injective ✓
surjective ✗
bijective ✗

(b)



function ✓
injective ✓
surjective ✓
bijective ✓

(c)



function ✓
injective ✗
surjective ✗
bijective ✗

Example 2

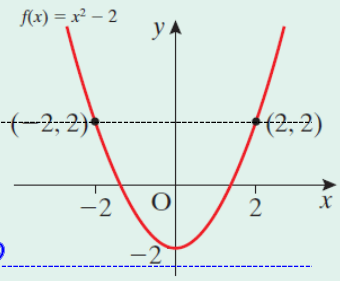
The graph of the function

$f: R \to R: x \to x^2 - 2$ is shown.

(i) What is the range of f?
(ii) Explain why f is not injective.
(iii) Explain why f is not surjective.
(iv) Suggest a domain for f to make the function injective.
(v) Suggest a codomain for f to make the function surjective.

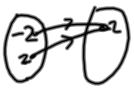
horizontal line test ①

horizontal line test ②



(i) Range $[-2, \infty)$

(ii) not injective because some outputs are shared with different inputs eg..



and horizontal line test ① crosses the curve in two places

(iii) not surjective, horizontal line test ② shows that not every element in codomain is in range

(iv) injective if $x > 0$, $x \in \mathbb{R}$

(v) codomain limited $f(x) > -2$