5th YearAlgebraChapter2Test2

Solutions

1. Solve the equation $x^2 - 6x + 5 = 0$.

Hence solve fully the equation

$$\left(t - \frac{6}{t}\right)^2 - 6\left(t - \frac{6}{t}\right) + 5 = 0.$$

Part 1

Solve: $x^2 - 6x + 5 = 0$

$(x - 5)(x - 1) = 0$

$\Rightarrow x = 5 \quad , x = 1$

Part 2

let $x = t - \frac{6}{t}$

When $x = 5$

$\Rightarrow 5 = t - \frac{6}{t}$

$5t = t^2 - 6$

$t^2 - 5t - 6 = 0$

$(t - 6)(t + 1) = 0$

$t = 6 \quad , t = -1$

When $x = 1$

$\Rightarrow 1 = t - \frac{6}{t}$

$1t = t^2 - 6$

$t^2 - t - 6 = 0$

$(t - 3)(t + 2) = 0$

$t = 3 \quad , t = -2$

There are 4 Solutions: $t = -2$, $t = -1$, $t = 3$, $t = 6$

2. If r_1 and r_2 are the roots of the equation $x^2 - \sqrt{3}x - 6 = 0$, evaluate $r_1 r_2$.

$$x^2 - \left(\begin{smallmatrix}\text{Sum}\\ \text{Roots}\end{smallmatrix}\right)x + \left(\begin{smallmatrix}\text{Product}\\ \text{Roots}\end{smallmatrix}\right) = 0$$

$\Rightarrow R_1 R_2 = -6$

[Product]

3. Find the range of values of k so that $kx^2 - 2kx - 3k - 12 = 0$ has real roots.

$\Delta = b^2 - 4ac \geq 0$ if Roots are Real

$a = k$

$b = -2k$

$c = -3k - 12$

$$\begin{aligned}\Delta &= (-2k)^2 - 4(k)(-3k-12)\\ &= 4k^2 + 12k^2 + 48k\\ &= 16k^2 + 48k\\ &= 16k(k+3) \geq 0\end{aligned}$$

Consider: If $k > 0 \Rightarrow \Delta > 0$

If $k = 0 \Rightarrow \Delta = 0$

If $k = -3 \Rightarrow \Delta = 0$

If k is negative, less than -3 ie $k < -3 \Rightarrow \Delta > 0$

If k is negative, between -3 and $0 \Rightarrow \Delta < 0$

Roots are real if $k \geq 0$ or $k \leq -3$

ie.. $0 \leq k \leq -3$

4. Show that $\dfrac{-1+\sqrt{3}}{1+\sqrt{3}} = 2-\sqrt{3}$.

$$\frac{-1+\sqrt{3}}{1+\sqrt{3}} = \frac{(-1+\sqrt{3})(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = \frac{-1+\sqrt{3}+\sqrt{3}-3}{1-3}$$

DOTS.

$$= \frac{-4+2\sqrt{3}}{2} = 2-\sqrt{3}$$

QED

5. Factorise fully $x^3 - x^2 - 14x + 24$.
Hence solve the equation $x^3 - x^2 - 14x + 24 = 0$.

Step 1: Guess to find Root & factor

let $f(x) = x^3 - x^2 - 14x + 24$

$$f(2) = (2)^3 - (2)^2 - 14(2) + 24$$
$$= 8 - 4 - 28 + 24 = 0$$

$\Rightarrow x = 2$ is a root & $(x-2)$ is a factor

Step 2: Divide to get quadratic factor

$$\begin{array}{r|l} & x^2 + x - 12 \\ \hline x-2 & \cancel{x^3} - x^2 - 14x + 24 \\ & \mp \cancel{x^3} \pm 2x^2 \\ \hline & \quad \cancel{x^2} - 14x \\ & \mp \cancel{x^2} \pm 2x \\ \hline & \qquad -12\cancel{x} + \cancel{24} \\ & \qquad \pm 12\cancel{x} \mp \cancel{24} \\ \hline \end{array}$$

Step 3: Factorise and solve the quadratic factor

$$x^2 + x - 12 = 0$$
$$(x+4)(x-3) = 0$$
$$\Rightarrow x = -4, \quad x = 3$$

ANSWERS: Factors: $(x-2)(x-3)(x+4)$
Solutions: $x = 2, \; x = 3, \; x = -4$

Find a cubic function for this curve

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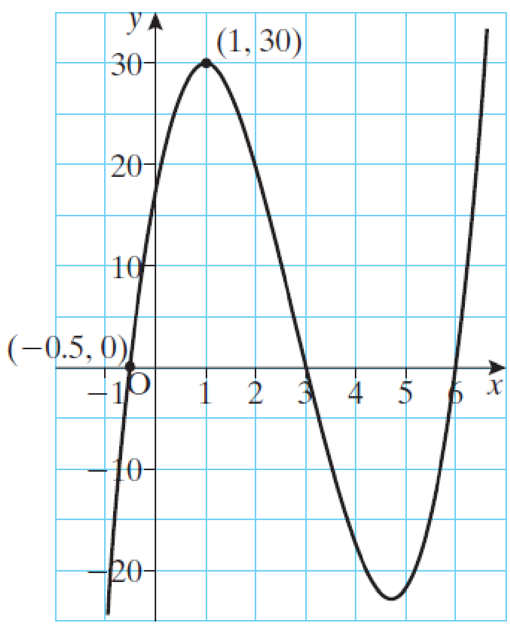

Roots : $x = -0.5$, $x = 3$, $x = 6$

factors $(x+\frac{1}{2})(x-3)(x-6)$

Function ?

$f(x) = k(x+\frac{1}{2})(x-3)(x-6)$

Contains $(1, 30)$

$\Rightarrow f(1) = k(1+\frac{1}{2})(1-3)(1-6) = 30$

$k(\frac{3}{2})(-2)(-5) = 30$

$15k = 30$

$k = 2$

$f(x) = 2(x+\frac{1}{2})(x-3)(x-6)$

expand

$= (2x+1)(x^2-6x-3x+18)$

$= (2x+1)(x^2-9x+18)$

$= 2x^3-18x^2+36x+x^2-9x+18$

ANSWER: $f(x) = 2x^3-17x^2+27x+18$