$\log_{\square}\square$ ← "The no." ; ↑ BASE

The log of a number is the power which the base must be raised to give you that number.

$2^3 = 8$ (POWER, BASE, ANSWER)

$3^{\square} = 81$ (? POWER, BASE, ANSWER)

Use logs to find the power eg.

$\log_3 81 = 4$

5. Express $\dfrac{4^2 \times 16^{\frac{1}{2}}}{64^{\frac{2}{3}} \times 4^3}$ in the form $4^n, n \in Z$.

$n = \log_{BASE}$ no.

$n = \log_4 \dfrac{4^2 \times 16^{1/2}}{64^{2/3} \times 4^3} = -2$

Key

$16 = 4^2$

$64 = 4^3$

$$\text{Expression} = \frac{(4^2)((4^2)^{\frac{1}{2}})}{(4^3)^{2/3}(4^3)}$$

$$= \frac{(4^2)(4^1)}{(4^2)(4^3)}$$

$$= \frac{4^3}{4^5}$$

$$= 4^{-2}$$

$n = -2$

6. Find the value of the rational number p for which $\dfrac{3^{\frac{1}{4}} \times 3 \times 3^{\frac{1}{6}}}{\sqrt{3}} = 3^p$.

$\log_{\text{BASE}}$ NUMBER = POWER

$p = \log_3 \dfrac{3^{1/4} \times 3 \times 3^{1/6}}{\sqrt{3}} = \dfrac{11}{12}$

$\dfrac{1}{4} + 1 + \dfrac{1}{6} = \dfrac{17}{12}$

$\dfrac{17}{12} - \dfrac{1}{2} = \dfrac{11}{12}$

$\dfrac{(3^{1/4})(3^1)(3^{1/6})}{3^{1/2}}$

$= \dfrac{3^{17/12}}{3^{1/2}} = 3^{11/12}$

$\Rightarrow p = \dfrac{11}{12}$

9. By multiplying the numerator and denominator by $(x-1)^{\frac{1}{2}}$, simplify

$$\frac{(x-1)^{\frac{1}{2}} + (x-1)^{-\frac{1}{2}}}{(x-1)^{\frac{1}{2}}}.$$

$= \dfrac{\sqrt{x-1} + \frac{1}{\sqrt{x-1}}}{\sqrt{x-1}}$

multiply by $\dfrac{\sqrt{x-1}}{\sqrt{x-1}}$

$= \dfrac{\sqrt{x-1}\sqrt{x-1} + \frac{1 \cdot \sqrt{x-1}}{\sqrt{x-1}}}{\sqrt{x-1}\sqrt{x-1}}$

$= \dfrac{x-1+1}{x-1}$

$= \dfrac{x}{x-1}$

9. By multiplying the numerator and denominator by $(x-1)^{\frac{1}{2}}$, simplify

$$\frac{(x-1)^{\frac{1}{2}} + (x-1)^{-\frac{1}{2}}}{(x-1)^{\frac{1}{2}}}.$$

multiply $\frac{(x-1)^{\frac{1}{2}}}{(x-1)^{\frac{1}{2}}}$

$$= \frac{(x-1)^{\frac{1}{2}}(x-1)^{\frac{1}{2}} + (x-1)^{-\frac{1}{2}}(x-1)^{\frac{1}{2}}}{(x-1)^{\frac{1}{2}}(x-1)^{\frac{1}{2}}}$$

$a^n a^m = a^{n+m}$

$$= \frac{(x-1)^1 + (x-1)^0}{(x-1)^1}$$

$$= \frac{x-1+1}{x-1}$$

$$= \frac{x}{x-1}$$