chapter 6

Length – Area – Volume

Section 6.3 3-Dimensional objects

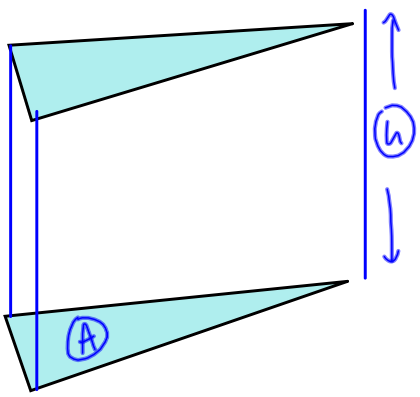
PROJECT MATHS

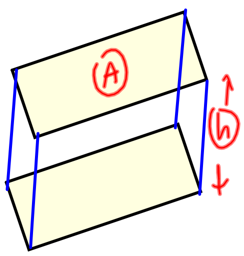
Text & Tests 6

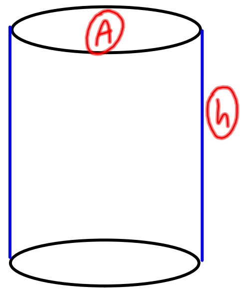
203

Prism

$V = Ah$

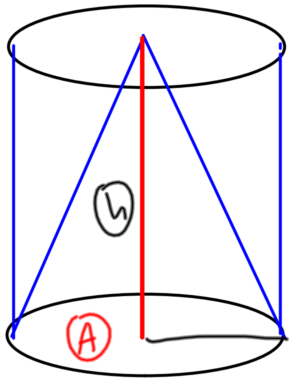




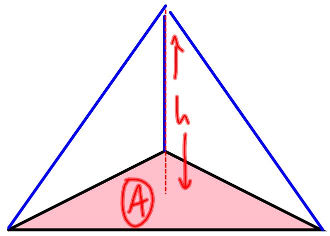


not prism because sides are curved.

Pyramid

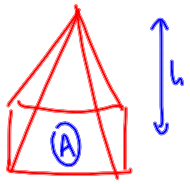


$$V = \frac{Ah}{3}$$



$$V = \frac{Ah}{3}$$

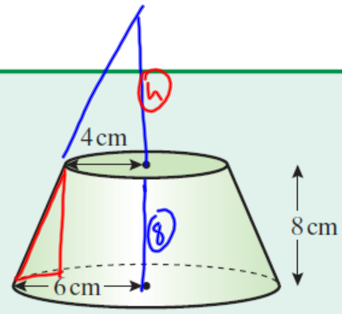
$V = Ah/3$

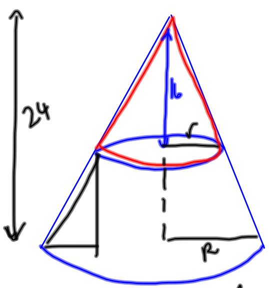


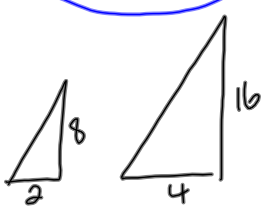
Example 1

Find the volume of the truncated cone shown (a frustum) correct to 1 place of decimals.

$$V_{cone} = \frac{\pi r^2 h}{3}$$







Similar triangles

Fustrum Volume?

$V =$ Big Cone $-$ Small Cone

$$V = \frac{\pi(6)^2(24)}{3} - \frac{\pi(4)^2(16)}{3}$$

$$V = 636.7\ cm^3$$

Example 2

A company makes ball bearings (spheres) for a machine with a diameter of 12 mm. They claim that they are produced to an accuracy of ±0.02 mm.
Find the largest and smallest ball bearing volumes produced.
Find the percentage error on (i) the diameter (ii) the volume.

Smallest diameter $= 12 - 0.02 = 11.98$ mm

$\Rightarrow$ radius $= \frac{11.98}{2} = 5.99$ mm

$V = \frac{4\pi R^3}{3}$ $\qquad V_{small} = \frac{4\pi(5.99)^3}{3} \approx 900.26 \text{ mm}^3$

Largest diameter $= 12 + 0.02 = 12.02$ mm

$\Rightarrow$ radius $= \frac{12.02}{2} = 6.01$ mm

$V_{large} = \frac{4\pi(6.01)^3}{3} \approx 909.31 \text{ mm}^3$

% Error on diameter?

Error $= \pm 0.02$ mm

'Correct' diameter $= 12$ mm

% Error diameter $= \frac{\pm 0.02 \times 100\%}{12} = \pm 0.17\%$

% Error on Volume?

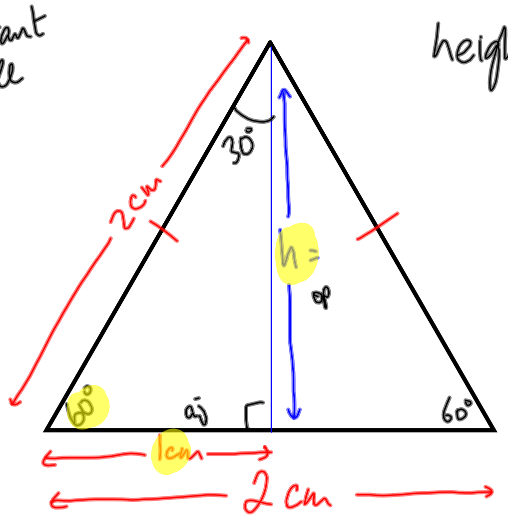
'Correct' Volume $= \frac{4\pi(6)^3}{3} \approx 904.78 \text{ mm}^3$

Error $= \pm(909.31 - 904.78) = \pm 4.53$

% Error Volume $= \frac{\pm 4.53 \times 100\%}{904.78} = \pm 0.5\%$

* Important triangle

height ?



Pythagoras $\Rightarrow$ $h^2 + 1^2 = 2^2$

$h^2 + 1 = 4$

$h^2 = 4 - 1 = 3$

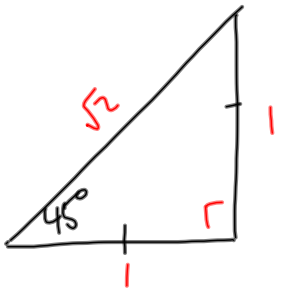
$h = \sqrt{3}$

OR

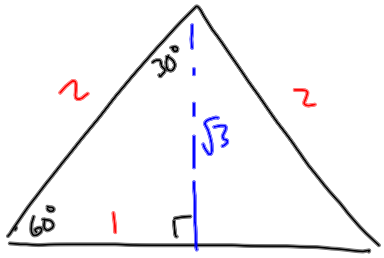
SOH
CAH
TOA

$\text{Tan } 60° = \frac{h}{1}$

$h = \text{Tan} 60° = \sqrt{3}$

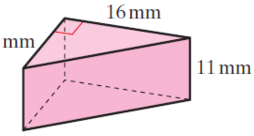


these should be known!

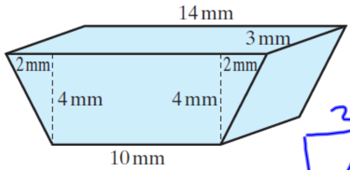


2. In a woodwork class, the students were asked to list in order from largest to smallest, the (i) volume (ii) **total** surface area of each of the following solid 3-dimensional objects, each answer given correct to the nearest whole number.

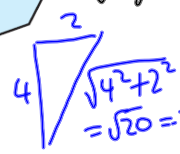
(h)



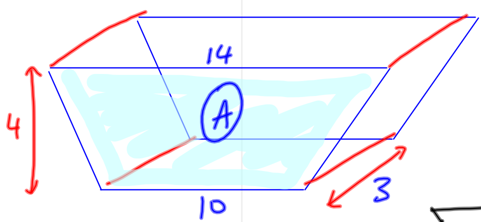
(i)



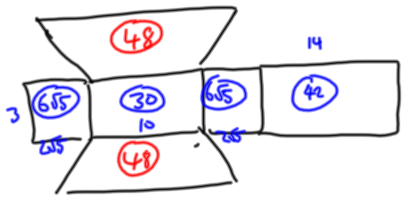
Pythagoras



$\sqrt{4^2+2^2} = \sqrt{20} = 2\sqrt{5}$



Surface Area?



Volume?

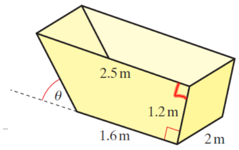
$\text{Area}_{\text{side}} = \left(\frac{10+14}{2}\right)4 = 48$

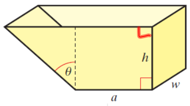
$V = Ah = 48(3) = 144 \text{ mm}^3$

$SA = 48+48+6\sqrt{5}+30+6\sqrt{5}+42$

$\approx 195 \text{ mm}^2$

3. This model of a skip is used by a recycling company.
   (i) Find the volume of the skip, correct to two places of decimals.
   (ii) The company offers a 'volume pick-up' at €80 per m³ or a 'weight pick-up' at €30 per 100 kg, assuming a full skip weighs 1.3 tonnes. Which option represents the best "value for money" for the customer?
   (iii) Write an equation for the volume of the skip in terms of a, h, w and θ.
   (iv) The recycling company wants to redesign the skip with a new angle $\theta = 45°$. If the width, height and overall volume must remain the same in order to fit on the truck, find, correct to one place of decimals, the new dimensions of the top and the bottom of the skip.





(i) Volume = $Ah = \underbrace{\left(\frac{2.5+1.6}{2}\right)(1.2)}_{\text{Trapezium}}\underbrace{(2)}_{h} = 4.92\,m^2$

(ii) Cost { Volume pick up = $80 \times 4.92 = €393.60$
Weight pick up = $(30)\left(\frac{1300}{100}\right) = €390.00$ ✓ Best value

(iii) Volume $= Aw$

$\tan\theta = \frac{x}{h}$

$x = h\tan\theta$

$\Delta = \frac{xh}{2} = \frac{(h\tan\theta)(h)}{2} = \frac{h^2\tan\theta}{2}$

$V = Aw = \left[ah + \frac{h^2\tan\theta}{2}\right]w = wah + \frac{wh^2\tan\theta}{2}$

(iv) given { $w = 2m$, $h = 1.2m$, $V = 4.92\,m^2$, $\theta = 45°$

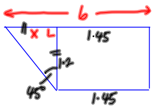
dimensions of top and bottom?
a = bottom

use formula from (iii)

$V = wah + \frac{wh^2\tan\theta}{2} \Rightarrow 4.92 = (2)(1.2)a + \frac{\cancel{(2)}(1.2)^2\tan 45°}{\cancel{2}}$

$\Rightarrow 4.92 = 2.4a + 1.44$

$2.4a = 3.48 \Rightarrow a = 1.45m$

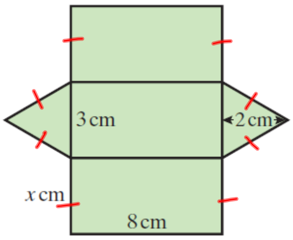


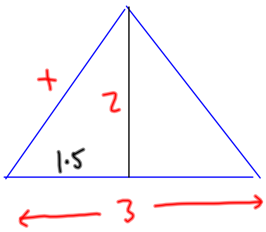
top dimension?

$b = 1.45 + x = 1.45 + 1.2$ (isosceles)

$b = 2.65m$

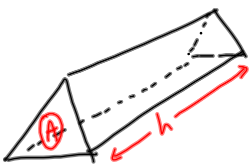
4. ✓ The net of a 3D figure is shown in the diagram.
Both triangles are isosceles and congruent.
   (i) Calculate the length of the side x cm. ✓
   (ii) Draw a sketch of the 3D figure and name it. ✓
   (iii) Calculate its volume. ✓
   (iv) Design a trapezoidal prism with the same volume.





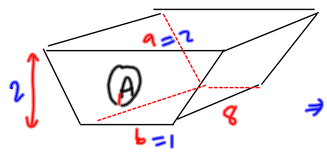
$x^2 = 2^2 + 1.5^2$

$x = \sqrt{4 + 1.5^2} = 5/2 = 2.5cm$



Triangular prism

$V = Ah \Rightarrow \left[\frac{3(2)}{2}\right]8 = 24cm^3$



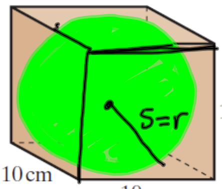
$V = 24\,cm^3$ if let $h = 8$

$\Rightarrow A = 3 = \left(\frac{a+b}{2}\right)2 \Rightarrow a + b = 3$

let $a = 2$, $b = 1$

5. (i) A student in a woodwork class is asked to fashion the largest sphere possible from the cube opposite. What volume of wood must be chipped away?

✓(ii) The student is then asked to calculate the volume of the smallest sphere that can enclose the cube fully.

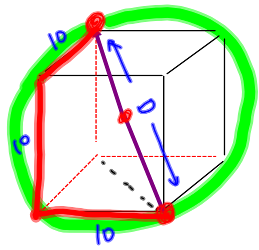


$$V_{Sphere} = \frac{4\pi r^3}{3} = \frac{4\pi(5)^3}{3} \approx 524\ cm^3$$

$$V_{CUBE} = 10^3 = 1000\ cm^3$$

$$V_{CHIPPED\ AWAY} \approx 1000 - 524 = 476\ cm^3$$

Pythagoras Theorem works in 3D!



$$D^2 = 10^2 + 10^2 + 10^2$$

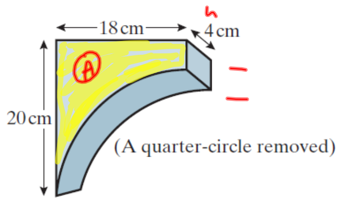
$$D^2 = 300$$

$$D = \sqrt{300} = 10\sqrt{3}\ cm$$

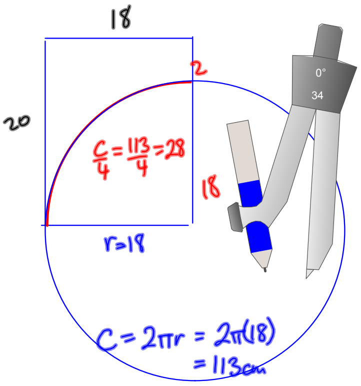
$$\Rightarrow r = \frac{10\sqrt{3}}{2} = 5\sqrt{3}\ cm$$

$$V_{SPHERE} = \frac{4\pi r^3}{3} = \frac{4\pi(5\sqrt{3})^3}{3} = 2720{\cdot}6$$

8. ✓ A steel support is to be made from a rectangular block of metal 4 cm thick, as shown. If a quarter-circle is removed, calculate the total surface area and the total volume of the support.



$$V = Ah$$

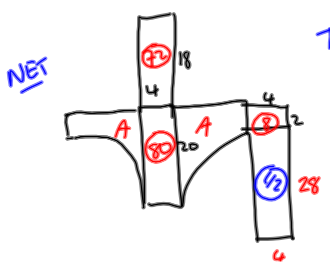


$$A_{rectangle} = LB = 18(20) = 360\ cm^2$$

$$A_{\frac{1}{4}circle} = \frac{\pi r^2}{4} = \frac{\pi(18)^2}{4} \approx 254\ cm^2$$

$$A = 360 - 254 = 106\ cm^2$$

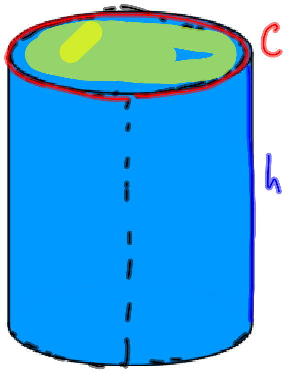
$$V = Ah = 106(4) = 424\ cm^2$$ ✓



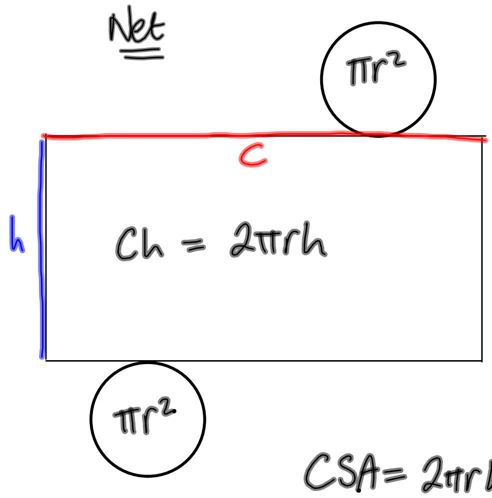
TOTAL Surface Area

$$= 2(106) + 72 + 80 + 8 + 112$$

$$= 484\ cm$$

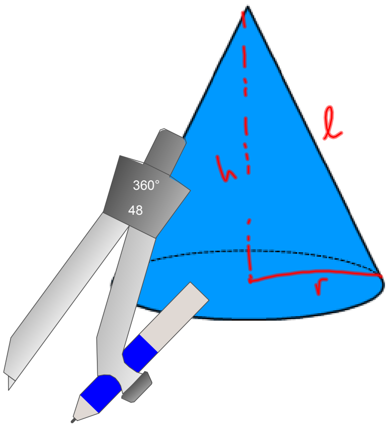


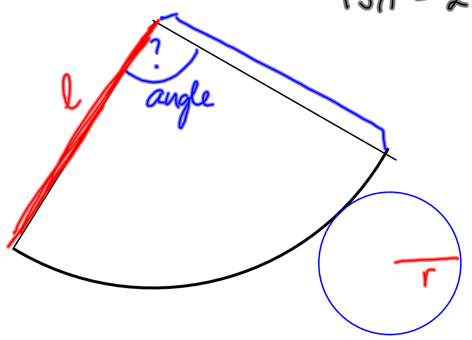
Net



$CSA = 2\pi rh$

$TSA = 2\pi rh + 2\pi r^2$

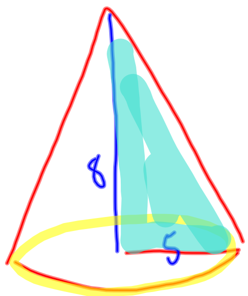


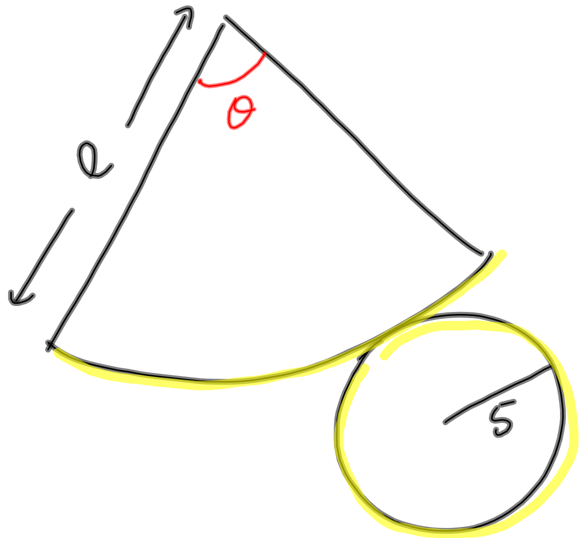


Make Net for a Cone

$h = 8$ cm

$r = 5$ cm

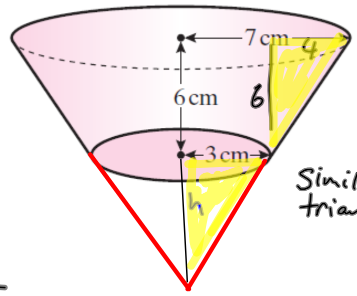




13. Find, correct to 1 place of decimals, the volume of this rubber stopper.

$$V = \frac{\pi r^2 h}{3}$$

Big Cone



Similar triangle

Fustrum = Big cone - Small Cone

$$h = \frac{6 \times 3}{4} = \frac{9}{2}$$