

Chapter 5 Financial Maths

Section 5.3 Instalment savings (annuities)

PROJECT MATHS Text & Tests 6

1. Instalment saving

Future value, F , of instalment savings over t instalments:

$$F = \text{€}P(1+i)^1 + \text{€}P(1+i)^2 + \text{€}P(1+i)^3 + \dots + \text{€}P(1+i)^t = \frac{\text{€}P(1+i)[(1+i)^t - 1]}{i}$$

$\text{€}P$ = the amount saved at the start of each month/year

t = the number of payments (months/years)

i = the interest rate, expressed as a decimal

$$S_n = \frac{a(1-r^n)}{1-r}$$

Example 1

Catriona saves €400 every three months for five years at an effective quarterly rate of 0.9%.

- Represent her savings by a geometric series
- Find the value of her investment at the end of the period.

$$5 \text{ years} = 20 \text{ quarters}$$

$$P = 400 \quad i = 0.9\% = 0.009 \quad 1+i = 1.009$$

$$F = P(1+i)^t$$

$$F_{20} = 400(1.009)^{20}$$

$$F_{19} = 400(1.009)^{19}$$

$$\vdots$$

$$F_2 = 400(1.009)^2$$

$$a = F_1 = 400(1.009)^1 \quad \left. \vphantom{a = F_1} \right\} \times 1.009 = r$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$r = 1.009$$

$$a = 403.60$$

$$n = 20$$

$$F_{\text{TOTAL}} = \frac{403.60(1-1.009^{20})}{1-1.009}$$

$$\approx \text{€}8,800.89$$