Section 4.6 Number patterns – Revisited

Sequence: 2, 4, 6, 8 .... Arithmetic $T_n = a + (n-1)d$

1st Difference +2 +2 +2 — 1st Diff. is Constant — same ⇒ LINEAR

Pattern: 1, 5, 13, 25

1st Difference +4 +8 +12

2nd Difference +4 +4

2nd Difference is constant ⇒ quadratic pattern.

PROJECT MATHS **Text & Tests 6**

	Pattern	To find a
1st difference constant	$T_n = an + b$	a = 1st difference
2nd difference constant	$T_n = an^2 + bn + c$	$2a$ = 2nd difference
3rd difference constant	$T_n = an^3 + bn^2 + cn + d$	$6a$ = 3rd difference

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$a = \frac{D}{1}$ $a = \frac{D}{2}$ $a = \frac{D}{6}$

not difference not T_1

Example 1

Express the nth term of the number pattern $-1, 13, 51, 125, 247, \ldots$ as a cubic polynomial.

Term	T_1	T_2	T_3	T_4	T_5
Pattern	-1	13	51	125	247
1st Diff.		14	38	74	122
2nd Diff.			24	36	48
3rd Diff.				12	12

3rd difference is = 12 which is constant

Cubic $\Rightarrow a = \frac{D}{6} = \frac{12}{6} = 2$

$\Rightarrow$ cubic pattern

Shape $\Rightarrow$ $an^3 + bn^2 + cn + d$

$a = 2 \Rightarrow 2n^3 + bn^2 + cn + d$

$n = 1$, $T_1 = -1 \Rightarrow 2(1)^3 + b(1)^2 + c(1) + d = -1$

$2 + b + c + d = -1$

$b + c + d = -3$ ①

$2n^3 + bn^2 + cn + d$

$b + c + d = -3$ ①

$n = 2$, $T_2 = 13$: $2(2)^3 + b(2)^2 + c(2) + d = 13$

$16 + 4b + 2c + d = 13$

$4b + 2c + d = -3$ ②

$n = 3$, $T_3 = 51$: $2(3)^3 + b(3)^2 + c(3) + d = 51$

$54 + 9b + 3c + d = 51$

$9b + 3c + d = -3$ ③

Eliminate ds

② $4b + 2c + d = -3$

$-$ ① $-b - c - d = +3$

$3b + c = 0$ ④

③ $9b + 3c + d = -3$

$-$ ① $-b - c - d = +3$

$8b + 2c = 0 \Rightarrow 4b + c = 0$ ⑤

Eliminate cs

⑤ $4b + c = 0$

$-$ ④ $-3b - c = 0$

$b = 0$

Sub $b = 0$ into ④: $3(0) + c = 0$, $c = 0$

Sub $b = 0$, $c = 0$ into ①: $0 + c + d = -3 \Rightarrow d = -3$

Formula for pattern: $2n^3 + 0n^2 + 0n - 3$ or $T_n = 2n^3 - 3$