Sequences –Series – Patterns

chapter 4

Section 4.5 Geometric series

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The sum to n terms of a geometric sequence,

$$S_n = \frac{a(1-r^n)}{1-r}, \quad \text{where } a \text{ is the first term and } r \text{ is the common ratio.}$$

Example 1

Find T_5 and S_5 of each of the following:

(i) $1 + 3 + 9 + \ldots$ (ii) $1 + \frac{1}{4} + \frac{1}{16} + \ldots$

Geometic

$a = 1$
$r = 3$
$T_5 = ?$
$n = 5$

$T_n = ar^{n-1}$

$S_5 = \frac{a(1-r^n)}{1-r}$

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \qquad S_5$
$1 + 3 + 9 + 27 + 81 = 121$

$\times 3$

$T_5 = 1(3)^{5-1} = 3^4 = 81$

$S_5 = \frac{1(1-3^5)}{1-3} = 121$

Example 2

In a geometric series, $T_3 = 32$ and $T_6 = 4$; find a and r and hence find S_8, the sum of the first eight terms.

$T_n = ar^{n-1}$

$T_3 = 32 \Rightarrow 32 = ar^{3-1}$

$\boxed{32 = ar^2}$ ①

$T_6 = 4 \Rightarrow 4 = ar^{6-1}$

$\boxed{4 = ar^5}$ ②

$\frac{②}{①}$ $\quad \frac{4}{32} = \frac{ar^5}{ar^2}$

$\frac{1}{8} = r^3 \Rightarrow r = \sqrt[3]{\frac{1}{8}}$

$\boxed{r = \frac{1}{2}} \Rightarrow 32 = a\left(\frac{1}{2}\right)^2$

$\Rightarrow \boxed{a = 128}$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_8 = \frac{128\left(1-\left(\frac{1}{2}\right)^8\right)}{\left(1-\frac{1}{2}\right)} = 255$