

**Example 4**

The product of the first three terms of a geometric sequence is 216 and their sum is 21. Given that the common ratio  $r$  is less than 1, find the first three terms of the sequence.

Let Sequence :  $a, ar, ar^2 \dots$

Product :  $(a)(ar)(ar^2) = 216$   
 $a^3 r^3 = 216$   
 $(ar)^3 = 216$   
 $ar = 6$  (1)  $\Rightarrow a = \frac{6}{r}$

Sum :  $a + ar + ar^2 = 21$   
 $a(1+r+r^2) = 21$  (2)

$\Rightarrow \frac{6}{r}(1+r+r^2) = 21$

$\times r \Rightarrow 6 + 6r + 6r^2 = 21r$   
 $6r^2 - 15r + 6 = 0$   
 $2r^2 - 5r + 2 = 0 \Rightarrow (2r-1)(r-2) = 0$   
 $r = \frac{1}{2}$  or  $r = 2$

$\Rightarrow a = \frac{6}{r} = \frac{6}{\frac{1}{2}} \Rightarrow a = 12$

Sequence:  $12, 6, 3 \dots$

**Example 5**

Find the number of terms in the geometric sequence  $81, 27, 9, \dots, \frac{1}{27}$ .

Sequence:  $81, 27, 9, 3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$  (T8)

$a = 81$

$r = \frac{T_2}{T_1} = \frac{27}{81} = \frac{1}{3}$

$T_n = ar^{n-1}$

$\frac{1}{27} = T_n \quad n=?$

$81 \left(\frac{1}{3}\right)^{n-1} = \frac{1}{27}$  ✓

$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{2187}$

BASE  $\rightarrow$   $\left(\frac{1}{3}\right)$       POWER  $\rightarrow$   $n-1$

$\log_{\frac{1}{3}} \left[\frac{1}{2187}\right] = 7 \Rightarrow n-1 = 7 \Rightarrow n = 8$

$\log_{\square} \square =$

Exponential sequences

Example 6

A ball is dropped from a height of 27 m and loses  $\frac{2}{3}$  of its height on each bounce.

- (i) Find the height of the ball on each of its first four bounces.
- (ii) Hence write down the height of the ball after the 10<sup>th</sup> bounce.
- (iii) After which bounce will the ball be at most 2.5 m above the ground?


T<sub>1</sub> happens after 1st Bounce

a = 9

r =  $\frac{1}{3}$

$T_n = ar^{n-1}$

$T_n = 9\left(\frac{1}{3}\right)^{n-1}$



(i) 1st 4 bounces :  $T_1$  9, 3, 1,  $\frac{1}{3}$   
(in meters)

(ii)  $T_{10} = 9\left(\frac{1}{3}\right)^9 \approx 0.00046 \text{ m}$   
 $\approx \frac{1}{2} \text{ mm}$

(iii) (from picture) after 3rd bounce  
the ball's height < 2.5m

7. The three numbers  $n - 2, n$  and  $n + 3$  are the first three terms of a geometric sequence. Find the value of  $n$  and hence write down the first four terms of the sequence.

Sequence:

a = n - 2

$r = \frac{T_{n+1}}{T_n}$

x(n)(n-2)

$n-2, n, n+3$   
 $T_1, T_2, T_3$

$r = \frac{n}{n-2} = \frac{n+3}{n}$

$\Rightarrow n^2 = (n+3)(n-2)$   
 $n^2 = n^2 - 2n + 3n - 6$   
 $n - 6 = 0 \Rightarrow n = 6$

$\Rightarrow r = \frac{n}{n-2} = \frac{6}{4} = \frac{3}{2}$

Sequence:

$4, 6, 9, 13.5$   
 $T_4$

$T_n = ar^{n-1} \Rightarrow T_n = 4\left(\frac{3}{2}\right)^{n-1}$

9. The first term of a geometric sequence is 16 and the fifth term is 9.  
What is the value of the seventh term?

$$a=16$$

$$T_n = ar^{n-1}$$

$$T_n = 16 \left(\frac{\sqrt{3}}{2}\right)^{n-1}$$

$$T_5 = 9, \quad T_7 = ?$$

$$T_5 = 16r^4 = 9$$

$$\Rightarrow r^4 = \frac{9}{16}$$

$$\Rightarrow r = \frac{\sqrt[4]{9}}{\sqrt[4]{16}} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow T_7 = 16 \left(\frac{\sqrt{3}}{2}\right)^6$$

$$T_7 = \frac{27}{4}$$