Example 4

The product of the first three terms of a geometric sequence is 216 and their sum is 21. Given that the common ratio r is less than 1, find the first three terms of the sequence.

T_1 a — T_2 ar — T_3 ar^2 (×r)

Product of 1st 3 terms $= 216$

$(a)(ar)(ar^2) = 216$

$a^3r^3 = 216 \Rightarrow (ar)^3 = 216$

$\Rightarrow ar = \sqrt[3]{216} \Rightarrow ar = 6$ ① $a = 6/r$

Sum of 1st 3 terms $= 21$

$a + ar + ar^2 = 21 \Rightarrow a(1 + r + r^2) = 21$ ②

Sub ① → ②

$\frac{6}{r}(1 + r + r^2) = 21$

$6 + 6r + 6r^2 = 21r$

$6r^2 - 15r + 6 = 0$

$2r^2 - 5r + 2 = 0$

$(2r - 1)(r - 2) = 0$

$2r - 1 = 0$

$r = \frac{1}{2}$ ✓ $r = 2$ ✗ $r < 1$

$r = \frac{1}{2} \Rightarrow a = \frac{6}{(\frac{1}{2})} = 12$

Sequence: 12, 6, 3, ...

Exponential sequences

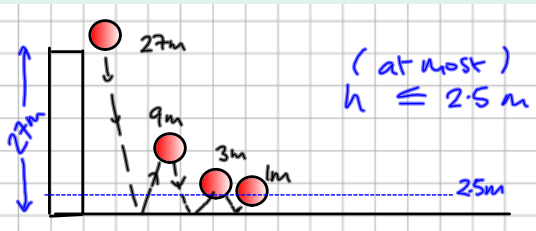
Example 6

A ball is dropped from a height of 27 m and loses $\frac{2}{3}$ of its height on each bounce.

(i) Find the height of the ball on each of its first four bounces.

(ii) Hence write down the height of the ball after the 10th bounce. $T_{11} = ?$

(iii) After which bounce will the ball be at most 2.5 m above the ground?



(at most) $h \leq 2.5$ m

T_1 27 — T_2 9 — T_3 3 — T_4 1

$a = 27$

$r = \frac{1}{3}$

$T_n = ar^{n-1}$

$T_{11} = 27(\frac{1}{3})^{11-1} = 27(\frac{1}{3})^{10} = \frac{1}{2187}$ m

After 3 bounces ball is lower than 2.5 m

Example 5

Find the number of terms in the geometric sequence $81, 27, 9, \ldots \frac{1}{27}$.

T_1 T_8

81 , 27 , 9 , 3 , 1 , 1/3 , 1/9 , 1/27

× 1/3

$T_n = ar^{n-1}$

$T_n = \frac{1}{27}$

$n = ?$

$a = 81$

$r = \frac{1}{3}$

$(81)\left(\frac{1}{3}\right)^{n-1} = \frac{1}{27}$

$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{2187}$

use logs

8. The third term of a geometric sequence is -63 and the fourth term is 189. Find

(i) the values of a and r

(ii) an expression for T_n.

T_1 T_2 T_3 T_4

−7 , 21 , −63 , 189

÷ −3 × −3

$r = \frac{T_4}{T_3}$

$r = \frac{189}{-63} = -3 = r$

$a = -7$

$T_n = ar^{n-1}$

$T_n = -7(-3)^{n-1}$