Sequences – Series – Patterns

chapter 4

Section 4.3 Arithmetic series

PROJECT MATHS

Text & Tests 6

$\cancel{S_n = \frac{n}{2}[2a + (n-1)]d}$

Example 1

Find the sum of the series 4 + 11 + 18 + 25 + ….. + 144.

T_n

$n = ?$

$S_n = \frac{n}{2}\left[2a + (n-1)\;d\right]$ Log Tables p.22

$a = 4$
$n = ?$
$d = 7$
$n = ?$

$T_n = a + (n-1)d$

$144 = 4 + (n-1)7 = 4 + 7n - 7$

$144 = 7n - 3$

$147 = 7n$

$n = \frac{147}{7} = 21$

$S_n = ?$

$S_n = \frac{21}{2}\left[2(4) + (22-1)\,7\right]$

$S_n = 1554$

Example 2

To celebrate the birth of his niece, an uncle offers to open a savings account with a deposit of €50. He also offers to every year add €10 more than he did the previous year until his niece is 21 years of age.

(i) Find an expression for S_n, the sum of money on deposit after n years.

(ii) Find S_{21}, the total saved after 21 years.

Series : $\underbrace{50 + 60}_{10} \underbrace{+ 70}_{10} + \ldots \ldots + T_n$

$a = 50$
$d = 10$
$n = 22$

S_n

$T_n = a + (n-1)d$

$T_n = 50 + 21(10) = 260$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{22}{2}[2(50) + (22-1)10] = 3410$

$\boxed{\text{Sum } 20} + \boxed{T_{21}}$

$= \boxed{\text{Sum } 21}$

$S_n - S_{n-1} = T_n$

Given S_n to find T_n :
$S_n - S_{n-1} = T_n$

$a = -3$
$d = 2$
$n = n$

Example 3

Given $S_n = n^2 - 4n$, find an expression for T_n and hence determine if the sequence is arithmetic.

$S_n = n^2 - 4n$

$S_1 = T_1 = a$

$S_2 = a + T_2$

$d = T_2 - T_1 = 2$

$n=1 \quad S_1 = T_1 = (1)^2 - 4(1) = -3$

$n=2 \quad S_2 = (2)^2 - 4(2) = -4$

$\Rightarrow -4 = -3 + T_2 \Rightarrow T_2 = -1$

$T_n = a + (n-1)d$

$T_n = -3 + (n-1)2$

$T_n = -3 + 2n - 2$

$\boxed{T_n = 2n - 5}$

Sigma (Σ) notation

$\sum$ = the sum of

$\sum_{r=1}^{4} T_r$ is the sum of $T_1 + T_2 + T_3 + T_4$

Example 5

(i) Use the sigma notation ($\sum$) to represent $2 + 6 + 10 + 14 + \ldots..$ for 45 terms.

(ii) For what value of n is $\sum_{r=1}^{n} 3r - 5 = 90$?

(iii) Find the value of $\sum_{r=1}^{8} 4r - 1$.

a $+4 = d$

(i) $T_n = a + (n-1)d = 2 + (n-1)4 = 2 + 4n - 4$

$T_n = 4n - 2$

$S_{45} = \sum_{r=1}^{45} (4r-2)$

(ii) $T_n = 3n - 5$

$S_n = 90$

$a = T_1 = 3(1) - 5$

$a = -2$

$n = ?$

$T_2 = 3(2) - 5$

$T_2 = 6 - 5$

$T_2 = 1$

$d = T_2 - T_1 = 1 - -2 = 3$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$90 = \frac{n}{2}[2(-2) + (n-1)3]$

$180 = n[-4 + 3n - 3]$

$180 = n[3n - 7]$

$180 = 3n^2 - 7n$

$3n^2 - 7n - 180 = 0$

$(3n + 20)(n - 9) = 0$

Reject

$n = 9$

3. How many terms of the series $5 + 8 + 11 + 14 + \ldots$ must be added to make a total of 98?

+3

$a = 5$
$d = +3$
* $S_n = 98$
$n = ?$

-198
$(\pm 28)(\mp 7)$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$98 = \frac{n}{2}\left[2(5) + (n-1)3\right]$$

$$196 = n\left[10 + 3n - 3\right]$$

$$196 = n\left[3n + 7\right]$$

$$196 = 3n^2 + 7n$$

$$3n^2 + 7n - 196 = 0$$

$$(3n + 28)(n - 7) = 0$$

Reject
$3n + 28 = 0$
$3n = -28$
$n = -9\frac{1}{3}$

$n = 7$ ✓