Sequences –Series – Patterns

chapter 4

Section 4.2 Arithmetic sequences

PROJECT MATHS Text & Tests 6

Example 1

Find the nth term (T_n) of the arithmetic sequence:

$-2, \quad 3, \quad 8, \quad 13, \ldots\ldots$

and hence find (i) T_{20} (ii) T_{21} (iii) $T_{21} - T_{20}$.

In every arithmetic sequence,
$T_1 = a$
$T_2 - T_1 = d$
$T_n - T_{n-1} = d$
$T_n = a + (n-1)d$

$a = -2$

$d = 5$

$T_n = a + (n-1)d$

$T_n = -2 + (n-1)5 = -2 + 5n - 5$

$T_n = 5n - 7$

$n = 20$ $\quad T_{20} = 5(20) - 7 = 93$

$n = 21$ $\quad T_{21} = 5(21) - 7 = 98$

$T_n - T_{n-1} = d$ $\quad T_{20} - T_{21} = 98 - 93 = 5 = d$

Example 2

Find the number of terms in the sequence

1, −3, −7, −11, ... −251.

$n = ?$

$a = 1$

$d = -4$

$T_n = -251$

$T_n = a + (n-1)d$

$-251 = 1 + (n-1)(-4)$

$-251 = 1 - 4n + 4$

$-251 = 5 - 4n$

$+256 = +4n$

$n = 64$

Example 3

In an arithmetic sequence, $T_4 = 6$ and $3T_2 = T_{10}$, find the values of a and d and hence write out the first 6 terms of the sequence.

$T_n = a + (n-1)d$

$n = 4$, $T_n = 6$ $\Rightarrow$

$T_4 = 6$

$6 = a + (4-1)d$

$6 = a + 3d$

$a + 3d = 6$ ①

$3T_2 = T_{10}$

$T_n = a + (n-1)d$

$T_2 = a + (2-1)d = a + d$

$T_2 = a + d$

$3T_2 = 3a + 3d$

$T_{10} = a + (10-1)d = a + 9d$

$T_{10} = a + 9d$

$3T_2 = T_{10}$

$3a + 3d = a + 9d$

$2a = 6d \Rightarrow a = 3d$ ②

② → ①

$3d + 3d = 6 \Rightarrow 6d = 6 \Rightarrow d = 1$

$a = 3(1) \Rightarrow a = 3$

Sequence : 3, 4, 5, 6, 7, 8 ....

Example 4

If $p + 2, 2p + 3$ and $5p - 2$ are three consecutive terms of an arithmetic sequence, find the value of $p, p \in R$.

$T_1 = p+2$

$T_2 = 2p+3$

$T_3 = 5p-2$

$T_2 - T_1 = d$

$T_3 - T_2 = d$

$T_2 - T_1 = T_3 - T_2$

$2p+3-(p+2) = 5p-2-(2p+3)$

$2p+3-p-2 = 5p-2-2p-3$

$p+1 = 3p-5$

$6 = 2p$

$p = 3$

Exercise 4.2

1. Find T_n, the nth term of the following arithmetic sequences. Hence find T_{22} for each sequence.

(i) 8, 13, 18, 23, ... (ii) 16, 36, 56, 76, ... (iii) 10, 7, 4, 1, ...

$a = 8$

$d = 5$

(i) 8, 13, 18, 23 ...

$T_n = a + (n-1)d$

$T_n = 8 + (n-1)5 = 8 + 5n - 5$

$T_n = 3 + 5n$

$n = 22$

$T_{22} = 3 + 5(22) = 3 + 110$

$T_{22} = 113$

3. Find the number of terms in each of the following arithmetic sequences:

(i) $-5, -1, 3, 7, \ldots\ldots 75$ (ii) $2, 5, 8, 11, \ldots\ldots 59$ (iii) $-\frac{3}{2}, -1, -\frac{1}{2}, 0, \ldots\ldots 14.$

$n = ?$
$a = -5$
$d = 4$

$T_n = a + (n-1)d$

If $T_n = 75$
$n = ?$

(i) $-5, -1, 3, 7 \ldots 75$

$T_n = -5 + (n-1)4 = -5 + 4n - 4$
$T_n = 4n - 9$

$75 = 4n - 9$
$84 = 4n$
$n = 21$

4. In an arithmetic sequence, $T_1 = 4$ and $T_7 = 22$. Using simultaneous equations, find

(i) the values of a and d (ii) the first five terms of the sequence (iii) T_{20}.

$a = T_1 = 4$

$T_n = a + (n-1)d$

$T_7 = 22 \Rightarrow$

T_1 T_7
$4, \cdot, \cdot, \cdot, \cdot, \cdot, 22$

$T_n = 4 + (n-1)d$

$22 = 4 + 6d$
$18 = 6d$
$d = 3$

Sequence: $4, 7, 10, 13, 16, 19 \ldots$

$d = 3 \Rightarrow$ $T_n = 4 + (n-1)3 = 4 + 3n - 3$
$T_n = 1 + 3n$

$n = 20 \Rightarrow$ $T_{20} = 1 + 3(20)$
$T_{20} = 61$