Complex numbers

chapter 3

Section 3.9 De Moivre's theorem

PROJECT MATHS
Text & Tests 6

If $z = r(\cos\theta + i\sin\theta)$, then using de Moivre's Theorem:

$$z^n = [r(\cos\theta + i\sin\theta)]^n$$
$$= r^n(\cos n\theta + i\sin n\theta) \quad \text{for all } n \in Z.$$

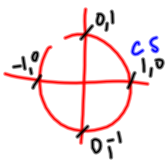
Example 1

Find the value of $\underbrace{\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)}_{z}{}^3$.

$z^n = r^n(\cos n\theta + i\sin n\theta)$

$r = 1$

$\theta = \frac{\pi}{6}$



$z^3 = 1^3\left(\cos \cancel{3}\left(\frac{\pi}{\cancel{6}\,2}\right) + i\sin \cancel{3}\left(\frac{\pi}{\cancel{6}\,2}\right)\right)$

$= 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ ✓

$= 1(0 + i1)$

$= i$

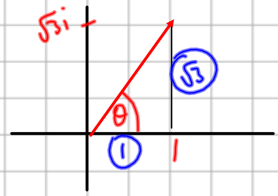
Example 2

Write $1 + \sqrt{3}i$ in polar form and hence find the value of $(1 + \sqrt{3}i)^9$.

$r = \sqrt{a^2 + b^2}$

$r = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\alpha = \tan^{-1}\left(\frac{|b|}{|a|}\right)$



$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

Polar form

$1 + \sqrt{3}i = 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$

de Moivre

$z^n = r^n\left(\cos(n\theta) + i\sin(n\theta)\right)$

$n = 9$

$(1+\sqrt{3}i)^9 = 2^9\left(\cos \frac{9\pi}{3} + i \sin \frac{9\pi}{3}\right)$

$= 512\left(\cos 3\pi + i \sin 3\pi\right)$

$= 512(-1 + i0)$

$= -512 + 0i$