Complex numbers

chapter 3

Section 3.9 De Moivre's theorem

$$[r\,\mathrm{Cis}\,\theta]^n = r^n\,\mathrm{Cis}\,n\theta$$

If $z = r(\cos\theta + i\sin\theta)$, then using de Moivre's Theorem:

$z^n = [r(\cos\theta + i\sin\theta)]^n$

$= r^n(\cos n\theta + i\sin n\theta)$ for all $n \in Z$.

Example 1

Find the value of $\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^3$.

$r = 1$

$\theta = \frac{\pi}{6}$

$(r\,\mathrm{Cis}\,\theta)^n = r^n\,\mathrm{Cis}\,n\theta$

$\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^3$

$= \cos\frac{3\pi}{6} + i\sin\frac{3\pi}{6}$

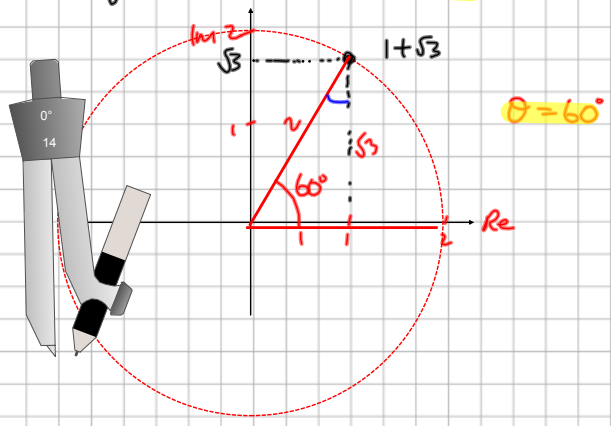
$= \mathrm{Cis}\,\frac{\pi}{2}$

Example 2

Write $1 + \sqrt{3}i$ in polar form and hence find the value of $(1 + \sqrt{3}i)^9$.

$r = \sqrt{a^2+b^2}$

$r = \sqrt{1^2+\sqrt{3}^2} = \sqrt{4} = 2 = r$



In Polar form

$1+\sqrt{3}i = 2\text{Cis}\,60°$

de Moivre

$(r\text{Cis}\theta)^n = r^n \text{Cis}\, n\theta$

$(1+\sqrt{3}i)^9 = (2\text{Cis}\,60°)^9 = 2^9\,\text{Cis}\,9(60°)$

$= 512\,\text{Cis}\,540°$

$= 512(-1+0i) = -512$

Use Calculator

Exercise 3.9

1. Use de Moivre's theorem to simplify each of the following, expressing your answers in the form $a + bi$:

(i) $\left(\cos\left(\frac{\pi}{8}\right) + i\sin\left(\frac{\pi}{8}\right)\right)^4$ (ii) $\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)^7$

de Moivre

$(r\text{Cis}\theta)^n = r^n\text{Cis}\,n\theta$

(i) $\left(\text{Cis}\,\frac{\pi}{8}\right)^4 = \text{Cis}\,\frac{4\pi}{8} = \text{Cis}\,\frac{\pi}{2}$

$= 0 + 1i$

Use calculator to change to rectangular form

(ii) $\left(\text{Cis}\,\frac{\pi}{6}\right)^7 = \text{Cis}\,\frac{7\pi}{6}$

$= -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

Use calculator to change to rectangular form

Exercise 3.9

1. Use de Moivre's theorem to simplify each of the following, expressing your answers in the form $a + bi$:

(iii) $\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)^8$ (iv) $\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^3$

de Moivre

$(r\text{Cis}\theta)^n = r^n\text{Cis}\,n\theta$

(iii) $\left(\text{Cis}\frac{\pi}{12}\right)^8 = \text{Cis}\frac{8\pi}{12} = \text{Cis}\frac{2\pi}{3}$

use calculator to change to rectangular form

$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

(iv) $\left(\text{Cis}\frac{2\pi}{3}\right)^3 = \text{Cis}\,3\left(\frac{2\pi}{3}\right) = \text{Cis}\,2\pi$

use calculator to change to rectangular form

$= 1 + 0i$