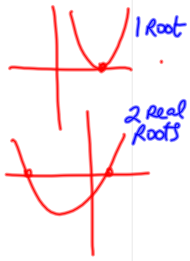
Complex numbers

Chapter 3

Section 3.6 Conjugate roots theorem



$ax^2+bx+c=0$

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

$x=\frac{-b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}$ | $x=\frac{-b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}$

if $\underbrace{x=d+ei \quad | \quad x=d-ei}_{\text{Conjugates}}$

Example 1

If $z=1+5i$ is a root of the equation $az^2+bz+c=0$, where $a,b,c\in R$, find the values of a,b,c.

If $z=1+5i$ is a Root
$\Rightarrow \bar{z}=1-5i$ is also a Root] Conjugate Root theorem

$x^2-[\text{sum Roots}]x+[\text{product Roots}]=0$

$x^2-[(1+\cancel{5i})+(1-\cancel{5i})]x+[(1+5i)(1-5i)]=0$

$x^2-2x+[1\pm25i^{\cancel{2}}]=0$

$x^2-2x+26=0$

Example 2

Given that $z = 2 + i$ is a root of $z^2 - 4z + 5 = 0$, show that $\overline{z}$ is also a root.

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$

$b = -4$

$c = 5$

$$z = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{4 \pm \sqrt{-4}}{2}$$

$$= \frac{4 \pm 2i}{2}$$

$$= 2 \pm i$$