Complex numbers

chapter 3

Section 3.6 Conjugate roots theorem

If $a+bi$ is a root of a quadratic then $a-bi$ is the other root.

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Example 2

Given that $z = 2 + i$ is a root of $z^2 - 4z + 5 = 0$, show that $\bar{z}$ is also a root.

$\bar{z}$ is the Conjugate of z

$z = 2 + i$
$\bar{z} = 2 - i$

Factor Theorem
If $f(\bar{z}) = 0$
then $\bar{z}$ is a root

$f(2-i) = ?$

$(2-i)^2 - 4(2-i) + 5 = ?$

$= 4 - 4i \mp 1\cancel{i^2} - 8 + 4i + 5$

$= \cancel{9} - \cancel{9} - \cancel{4i} + \cancel{4i}$

$= 0$

$\therefore$ $\bar{z}$ is a root.

Conjugate roots theorem

If z is a root of $az^n + bz^{n-1} + \ldots. dz + c = 0$ where $a, b, c, d, \ldots.$ are all $\in R$, then $\bar{z}$ is also a root of this equation.

Example 3

Given that $z = 1 + 2i$ is a root of $z^3 - z^2 + 3z + 5 = 0$, show that $\bar{z}$ is also a root. Hence find the third root.

Factor Theorem

If $\bar{z}$ is a root then $f(\bar{z}) = 0$

$\bar{z} = 1 - 2i$

the Conjugate of z

$f(1-2i) = ?$

$= (1-2i)^3 - (1-2i)^2 + 3(1-2i) + 5$

note: $\bar{z}^3 = (\bar{z}^2)(\bar{z})$

$= (1-4i+4i^2)(1-2i) - (1-4i+4i^2) + 3 - 6i + 5$

$= (-3-4i)(1-2i) - (-3-4i) + 8 - 6i$

$= -3 + 6i - 4i + 8i^2 + 3 + 4i + 8 - 6i$

$= \cancel{-11} - \cancel{10i} + \cancel{11} + \cancel{10i}$

$= 0$

$\therefore$ $1-2i$ is a root.

Conjugate roots theorem

If z is a root of $az^n + bz^{n-1} + \ldots. dz + c = 0$ where $a, b, c, d, \ldots.$ are all $\in R$, then $\bar{z}$ is also a root of this equation.

Example 3

Given that $z = 1 + 2i$ is a root of $z^3 - z^2 + 3z + 5 = 0$, show that $\bar{z}$ is also a root. Hence find the third root.

① Get quadratic factor

$z = 1 + 2i$ root

$\Rightarrow \bar{z} = 1 - 2i$ root

Quadratic from roots:

$z^2 - \left(\begin{smallmatrix}\text{Sum}\\\text{Roots}\end{smallmatrix}\right)z + \left(\begin{smallmatrix}\text{Product}\\\text{Roots}\end{smallmatrix}\right) = 0$

$z + \bar{z} = 2$

$z.\bar{z} = 1^2 + 2^2 = 5$

② Divide quadratic into cubic

③ turn factor into root.

Quadratic factor

$\Rightarrow z^2 - 2z + 5 = 0$

$$\begin{array}{r}
z + 1 \\
z^2 - 2z + 5 \enclose{longdiv}{\cancel{z^3} - z^2 + 3z + 5} \\
-\cancel{z^3} + 2z^2 - 5z \\
\hline
\cancel{z^2} - \cancel{2z} + \cancel{5} \\
-\cancel{z^2} + \cancel{2z} - \cancel{5} \\
\hline
0
\end{array}$$

$\therefore \bar{z}$ is a root

$\Rightarrow z + 1$ is other factor

$\Rightarrow z = -1$ is root

2. Solve these equations, giving your answers in the form $a \pm bi, a, b \in R$.

(i) $z^2 - 2z + 17 = 0$ (ii) $z^2 + 4z + 7 = 0$

(i)

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$
$b = -2$
$c = 17$

$$z = \frac{+2 \pm \sqrt{(-2)^2 - 4(1)(17)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-64}}{2}$$

$$= \frac{2 \pm 8i}{2}$$

$$= 1 \pm 4i$$

6. Given that $2 + 3i$ is one root of the equation $2z^3 - 9z^2 + 30z - 13 = 0$, find the other two roots.

① Get imaginary root

Conjugate Root Theorem

If $2+3i$ is a root so is $\boxed{2-3i}$

② Form Quadratic factor

$x^2 - \underset{\text{Roots}}{\text{Sum}}\, x + \underset{\text{Roots}}{\text{Product}} = 0$

Sum $= 4$
Product $= 2^2 + 3^2 = 13$

Quadratic factor
$z^2 - 4z + 13 = 0$

③ Divide by quadratic to get linear factor

$$\begin{array}{r} 2z - 1 \\ z^2 - 4z + 13 \,\big)\, 2z^3 - 9z^2 + 30z - 13 \\ \mp 2z^3 \pm 8z^2 \mp 26z \\ \hline -z^2 + 4z - 13 \\ \pm z^2 \mp 4z \pm 13 \\ \hline \end{array}$$

The other factor is $2z - 1$

④ Write real root from factor

$\Rightarrow \boxed{z = \frac{1}{2}}$ is real root