

11. If $(x + iy)^2 = 8 - 6i$, find the values of x and y , $x, y \in \mathbb{R}$.

expand LHS $(a+b)^2 = a^2 + 2ab + b^2$ $\text{Re} = \text{Re}$ $\text{Im} = \text{Im}$	$\begin{aligned} (x+iy)^2 &= 8-6i \\ x^2 + 2xyi + y^2 i^2 &= 8-6i \\ x^2 - y^2 + 2xyi &= 8-6i \end{aligned}$ $\begin{aligned} x^2 - y^2 &= 8 \quad \textcircled{1} \\ 2xy &= -6 \\ xy &= -3 \\ x &= -3/y \quad \textcircled{2} \end{aligned}$
$\text{Solve simultaneous equa:}$ $\text{Sub } \textcircled{1} \text{ into } \textcircled{2}$	$\begin{aligned} (-3/y)^2 - y^2 &= 8 \\ 9/y^2 - y^2 &= 8 \\ 9 - y^4 &= 8y^2 \\ y^4 + 8y^2 - 9 &= 0 \\ (y^2 + 9)(y^2 - 1) &= 0 \end{aligned}$
$\text{multiply by } y^2$ factorise	$\begin{aligned} y^2 &= -9 \\ y &= \pm \sqrt{-9} \\ y &= \pm 3i \end{aligned}$
get X values $x = -3/y$	$\begin{aligned} y &= 3i & x &= -\frac{3}{3i} = -\frac{1}{i} = -\frac{1(-i)}{i(-i)} = \frac{i}{1} = i \\ y &= -3i & x &= -\frac{3}{-3i} = \frac{1}{i} = \frac{1(-i)}{i(-i)} = \frac{-i}{1} = -i \\ y &= 1 & x &= -\frac{3}{1} = -3 \\ y &= -1 & x &= -\frac{3}{-1} = 3 \end{aligned}$
$\text{pairs of solutions}$	$\begin{aligned} x &= i, y = 3i \\ x &= -i, y = -3i \\ x &= -3, y = 1 \\ x &= 3, y = -1 \end{aligned}$