Example 3

Given that $(z + 1)(2 - i) = 3 - 4i$, find z in the form $x + yi$, where $x, y \in R$.

let $z = x + yi$

Rewrite $\Rightarrow$ $(x + yi + 1)(2 - i) = 3 - 4i$

expand LHS $2(x + yi + 1) - i(x + yi + 1) = 3 - 4i$

$2x + 2yi + 2 - ix - yi^2 - i = 3 - 4i$

Re = Re $2x + 2 + y = 3 \Rightarrow \boxed{2x + y = 1}$ ①

Im = Im $2y - x - 1 = -4 \Rightarrow \boxed{-x + 2y = -3}$ ②

Solve: ① $2x + y = 1$

$+2$ ② $-2x + 4y = -6$

$5y = -5 \Rightarrow y = -1$

Sub $y = -1$ into ① $2x - 1 = 1$

$2x = 2 \Rightarrow x = 1$

$z = x + yi \Rightarrow \boxed{z = 1 - 1i}$

Example 4

Express $\sqrt{5 + 12i}$ in the form of $a + bi$, where $a, b \in R$.

let $\sqrt{5 + 12i} = a + bi$

Square $(a+b)^2 = a^2 + 2ab + b^2$

$5 + 12i = (a + bi)^2$

$= a^2 + 2abi + b^2 i^2$

$5 + 12i = a^2 + 2abi - b^2$

Re = Re $\boxed{5 = a^2 - b^2}$ ①

Im = Im $12 = 2ab \Rightarrow \boxed{6 = ab}$ ② $\Rightarrow a = \frac{6}{b}$

Solve — Rewrite ② Sub into ① $5 = \left(\frac{6}{b}\right)^2 - b^2$

$5 = \frac{36}{b^2} - b^2$

$\times b^2$ $5b^2 = 36 - b^4 \Rightarrow b^4 + 5b^2 - 36 = 0$

Solve degree 4 equation like a quadratic — FACTORISE $(b^2 + 9)(b^2 - 4) = 0$

$b^2 = -9$, $b^2 = 4$

$b = \pm\sqrt{-9}$, $b = \pm\sqrt{4}$

SOLUTIONS $b = \pm 3i$ ✗, $b = \pm 2$ ✓ Since $b \in R$

Sub $b = \pm 2$ into ② $\Rightarrow a = 6/+2 \Rightarrow a = 3$

$a = 6/-2 \Rightarrow a = -3$

TWO ANSWERS POSSIBLE either $\sqrt{5 + 12i} = 3 + 2i$ or $\sqrt{5 + 12i} = -3 - 2i$

5. Simplify each of the following.

(i) $\dfrac{(3+4i)+(2+i)}{4-i}$ (ii) $\dfrac{(2-6i)-(3+2i)}{2+2i}$

Simplify numerator

$$\frac{2-6i-3-2i}{2+2i} = \frac{-1-8i}{2+2i}$$

Rationalise denominator DOTS

$$\frac{(-1-8i)(2-2i)}{(2+2i)(2-2i)}$$

$$= \frac{-2+2i-16i+16i^2}{2^2+2^2}$$

$$= \frac{-18-14i}{8}$$

$$= \frac{-9-7i}{4}$$

$$= -\frac{9}{4} - \frac{7}{4}i$$

6. Find the values of x and y in each of the following:

(iii) $x+yi = \dfrac{7+i}{2-i}$ (iv) $x+yi = (2-3i)^2$

Simplify RHS DOTS

$$\frac{(7+i)(2+i)}{(2-i)(2+i)}$$

$$= \frac{14+7i+2i+1i^2}{2^2+1^2}$$

$$= \frac{13+9i}{5}$$

$$= \underbrace{\frac{13}{5}}_{x} + \underbrace{\frac{9}{5}}_{y}i$$