9. If $z_1 = -3 + 4i$ and $z_2 = 1 + 2i$ are two complex numbers and $z_1 + (p + iq)z_2 = 0$ where $p, q \in R$, find the values of p and q.

Hint: $0 = 0 + 0i$

FOIL

$(-3+4i) + \underbrace{(p+iq)(1+2i)} = 0 + 0i$

$-3 + 4i + p + 2pi + qi + 2q\cancel{i^2} = 0 + 0i$

$\underbrace{(-3 + p - 2q)}_{Re} + \underbrace{(4i + 2pi + qi)}_{Im} = 0 + 0i$

$\boxed{\begin{aligned} Re &= Re \\ Im &= Im \end{aligned}}$

$-3 + p - 2q = 0$

$p - 2q = 3$ ①

$(4 + 2p + q)i = 0i$

$4 + 2p + q = 0$

$2p + q = -4$ ②

② − 2①

$\cancel{2p} + q = -4$

$-\cancel{2p} + 4q = -6$

$5q = -10 \Rightarrow q = -2$

→ ① $p - 2(-2) = 3 \Rightarrow p + 4 = 3 \Rightarrow p = -1$

10. Given $z = \sqrt{3 + 4i}$, find z in the form of $a \pm bi$, where $a, b \in R$.

$z = \sqrt{3+4i}$ $\quad z = a + bi$ $\quad (a+b)^2 = a^2 + 2ab + b^2$

$\Rightarrow$ let $\sqrt{3+4i} = a + bi$

square $3+4i = (a+bi)^2$

$= a^2 + 2abi \mp b^2 \cancel{i^2}$

$3+4i = \underbrace{(a^2 - b^2)}_{Re} + \underbrace{(2abi)}_{Im}$ $\quad$ Re = Re, Im = Im

$3 = a^2 - b^2$ $\quad | \quad$ $4i = 2abi$

$4 = 2ab$

$2 = ab \Rightarrow a = \frac{2}{b}$

$\Rightarrow 3 = \left(\frac{2}{b}\right)^2 - b^2$

$\Rightarrow 3 = \frac{4}{b^2} - b^2 \quad \xrightarrow{\times b^2} \quad 3b^2 = 4 - b^4$

$\Rightarrow b^4 + 3b^2 - 4 = 0$

$(b^2 + 4)(b^2 - 1) = 0$

$b^2 = -4$ or $b^2 = 1$

$b = \mp\sqrt{-4}$ or $b = \pm 1$ ✓

$b = \pm 2i$ ✗

$a = \frac{2}{b} \Rightarrow b = 1 \Rightarrow a = \frac{2}{1} = 2$

$b = -1 \Rightarrow a = \frac{2}{-1} = -2$

$z = a \pm bi$ $\quad$ either $z = 2 + 1i$

or $z = -2 - 1i$