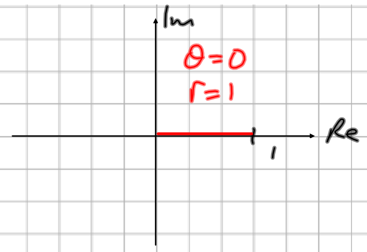
10. Use de Moivre's theorem to find, in polar form, the five roots of the equation $z^5 = 1$. Choose one of the roots w, where $w \neq 1$, and prove that $w^2 + w^3$ is real.

Complex form $z^5 = 1 + 0i$

polar form

$r \operatorname{Cis} \theta$



$z_1^5 = 1 \operatorname{Cis} 0°$
$z_2^5 = 1 \operatorname{Cis} 360°$
$z_3^5 = 1 \operatorname{Cis} 720°$
$z_4^5 = 1 \operatorname{Cis} 1080°$
$z_5^5 = 1 \operatorname{Cis} 1440°$

de Moivre

$(r \operatorname{Cis} \theta)^n = r^n \operatorname{Cis} n\theta$

$z_1 = (1 \operatorname{Cis} 0°)^{1/5} = \operatorname{Cis} 0°$
$= 1 + 0i$

$z_2 = (1 \operatorname{Cis} 360°)^{1/5} = \operatorname{Cis} 72°$
$= 0.31 + 0.95i$

$z_3 = (1 \operatorname{Cis} 720°)^{1/5} = \operatorname{Cis} 144°$
$= -0.81 + 0.59i$

$z_4 = (1 \operatorname{Cis} 1080°)^{1/5} = \operatorname{Cis} 216°$
$= -0.81 - 0.59i$

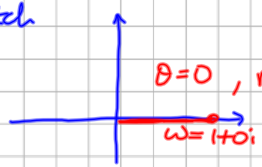
$z_5 = (1 \operatorname{Cis} 1440°)^{1/5} = \operatorname{Cis} 288°$
$= 0.31 - 0.95i$

What is the cube Root of unity?

$z^3 = 1$, find 3 values of z.

let $w = 1 + 0i$

sketch



polar form
$r \operatorname{Cis} \theta$

$\omega_1 = 1 \operatorname{Cis} 0°$
$\omega_2 = 1 \operatorname{Cis} 360°$
$\omega_3 = 1 \operatorname{Cis} 720°$

$z = \omega^{1/3}$

$z_1 = (\operatorname{Cis} 0°)^{1/3} = \operatorname{Cis} 0°$
$= 1 + 0i$

$z_2 = (\operatorname{Cis} 360)^{1/3} = \operatorname{Cis} 120°$
$= -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$z_3 = (\operatorname{Cis} 720)^{1/3} = \operatorname{Cis} 240°$
$= -\frac{1}{2} - \frac{\sqrt{3}}{2} i$