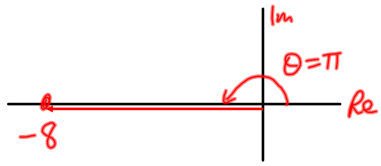
5. Find the values of z for which $z^3 = -8$, giving your answer in $a + bi$ form.

$z = (-8)^{\frac{1}{3}}$

let $w = -8$



$r = 8$
$\theta = \pi$

Polar form: $w = 8(\text{Cis}\ \pi)$

General Polar form: $w = 8(\text{Cis}\ (\pi + 2n\pi))$

$z = w^{1/3}$: $z = [8(\text{Cis}\ (\pi + 2n\pi))]^{1/3}$

de moivre

$z^n = r^n(\text{Cis}\ n\pi)$: $= 8^{1/3}(\text{Cis}\ \frac{1}{3}(\pi + 2n\pi))$

$= 2\left(\text{Cis}\left(\frac{\pi + 2n\pi}{3}\right)\right)$

let $n = 0, 1, 2$

$n=0$: $z_0 = 2\left(\text{Cis}\left(\frac{\pi}{3}\right)\right) = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 1 + \sqrt{3}i$

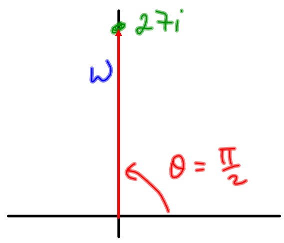
$n=1$: $z_1 = 2\left(\text{Cis}\left(\frac{\pi + 2\pi}{3}\right)\right) = 2(\text{Cis}\ \pi) = 2(-1 + 0i) = -2$

$n=2$: $z_2 = 2\left(\text{Cis}\left(\frac{\pi + 4\pi}{3}\right)\right) = 2\left(\text{Cis}\ \frac{5\pi}{3}\right) = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 1 - \sqrt{3}i$

8. Find the cube roots of $27i$.

$z^3 = 27i$
$z = ?$
$z = (27i)^{1/3}$

let $w = 27i = z^3$



$r = 27$
$\theta = \frac{\pi}{2}$

$\text{Cis}\,\theta = \cos\theta + i\sin\theta$

Polar form: $w = 27(\text{Cis}\ \frac{\pi}{2})$

General: $w = 27(\text{Cis}\ (\frac{\pi}{2} + 2n\pi))$

$z = w^{1/3}$: $z = [27(\text{Cis}\ (\frac{\pi}{2} + 2n\pi))]^{1/3}$

de moivre: $= 27^{1/3}(\text{Cis}\ \frac{1}{3}(\frac{\pi}{2} + 2n\pi))$

$= 3(\text{Cis}\ \frac{1}{3}(\frac{\pi}{2} + 2n\pi))$

let $n = 0, 1, 2$

$n=0$: $z_0 = 3(\text{Cis}\ \frac{\pi}{6}) = 3\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \frac{3\sqrt{3}}{2} + \frac{3}{2}i$

$n=1$: $z_1 = 3(\text{Cis}\ \frac{1}{3}(\frac{\pi}{2} + 2\pi)) = 3(\text{Cis}\ \frac{5\pi}{6})$

$= 3\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$

$n=2$: $z_2 = 3(\text{Cis}\ \frac{1}{3}(\frac{\pi}{2} + 2(2)\pi)) = 3(\text{Cis}\ \frac{3\pi}{2})$

$= 3(0 - 1i) = -3i$