Complex numbers

chapter 3

Section 3.10 Applications of de Moivre's Theorem

PROJECT MATHS Text & Tests 6

How to find the *n*th root of a complex number

Given $z = a + bi$,
then $z = r[\cos(\theta + 2n\pi) + i\sin(\theta + 2n\pi)]$
where $n \in N$ is the **general polar form** of z.

Example 3

Solve the equation $z^3 = 8i$.

Solve a cubic equation → 3 solutions

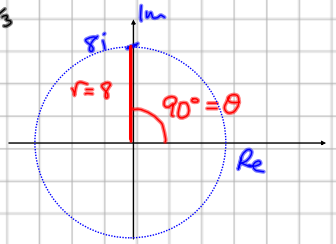
$z = ?$ Cube root

$rCiS\theta$ = polar form

3 angles that give solutions:
$\theta = 90°$
$\theta_1 = 360° + 90° = 450°$
$\theta_2 = 2(360°) + 90° = 810°$

$(rCiS\theta)^n = r^n CiS n\theta$

$z = (8i)^{1/3}$ (ω)



POLAR FORM: $\omega = 8i = 8\,CiS\,90°$
GENERAL POLAR FORM: $8\,CiS\,(90° + 360°n)$

$z_1 = (8\,CiS\,90°)^{1/3} = 8^{1/3}\,CiS\,\frac{1}{3}(90°) = 2\,CiS\,30°$ $= \sqrt{3} + i$

$z_2 = (8\,CiS\,450°)^{1/3} = 8^{1/3}\,CiS\,\frac{1}{3}(450°) = 2\,CiS\,150°$ $= -\sqrt{3} + i$

$z_3 = (8\,CiS\,810°)^{1/3} = 8^{1/3}\,CiS\,\frac{1}{3}(810°) = 2\,CiS\,270°$ $= -2i$

8. Find the cube roots of $27i$.

① $z^3 = w \Rightarrow z = w^{1/3}$ write w in $rCiS\theta$ form

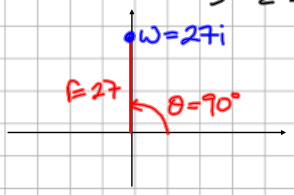
② Work out 3 arguments
$\theta_0 = 90°$
$\theta_1 = 90 + 360 = 450°$
$\theta_2 = 90 + 2(360) = 810°$

③ apply deMoivre's theorem
$(rCiS\theta)^n = r^n CiS\, n\theta$
to 3 versions of polar complex number

④ evaluate with calculator

Solve $z^3 = 27i \Rightarrow z = (27i)^{1/3}$

let $w = 27i$



polar form: $27CiS\,90°$
or $27CiS\,450°$
or $27CiS\,810°$

$z_1 = (27\,CiS\,90°)^{1/3} = 3\,CiS\,30°$
$= 3(\sqrt{3}/2 + 1/2\, i) = 3\sqrt{3}/2 + 3/2\, i$

$z_2 = (27\,CiS\,450°)^{1/3} = 3\,CiS\,150°$
$= 3(-\sqrt{3}/2 + 1/2\, i) = -3\sqrt{3}/2 + 3/2\, i$

$z_3 = (27\,CiS\,810°)^{1/3} = 3\,CiS\,270°$
$= 3(0 - 1i) = -3i$