

Exercise 2.9

1. Show that $(x - 3)$ is a factor of $x^2 - 8x + 15$.

if $(x-3)$ is
factor
 $\Rightarrow x=3$ is
solution
 $\Rightarrow f(3)=0$

$$\begin{aligned}f(3) &= (3)^2 - 8(3) + 15 \\ &= 9 - 24 + 15 \\ &= 24 - 24 \\ &= 0\end{aligned}$$

2. Show that $(x - 1)$ is a factor of $x^3 - x^2 - 9x + 9$.

$x=1$ is soln
 $f(1)=0$

$$\begin{aligned}(1)^3 - (1)^2 - 9(1) + 9 \\ &= 1 - 1 - 9 + 9 \\ &= 0\end{aligned}$$

3. Show that $(x + 2)$ is a factor of $x^3 + 6x^2 + 11x + 6$.

$$\begin{aligned}x &= -2 \text{ is} \\ \text{Soln} \\ f(-2) &= 0\end{aligned}$$

$$\begin{aligned}(-2)^3 + 6(-2)^2 + 11(-2) + 6 \\ &= -8 + 24 - 22 + 6 \\ &= -30 + 30 \\ &= 0\end{aligned}$$

4. Show that $(x - 2)$ is a factor of $2x^3 - 3x^2 - 12x + 20$.

$$\begin{aligned}\text{Soln } x &= 2 \\ f(2) &= 0\end{aligned}$$

$$\begin{aligned}2(2)^3 - 3(2)^2 - 12(2) + 20 \\ &= 16 - 12 - 24 + 20 \\ &= 36 - 36 \\ &= 0\end{aligned}$$

5. Investigate if $(x - 2)$ is a factor of $x^3 - 5x^2 + 8x - 4$.

$$f(2) = 0$$

$$\begin{aligned} & (2)^3 - 5(2)^2 + 8(2) - 4 \\ &= 8 - 20 + 16 - 4 \\ &= -24 + 24 \\ &= 0 \quad \text{Yes is factor} \end{aligned}$$

6. Show that $(2x - 1)$ is a factor of $2x^3 + 7x^2 + 2x - 3$.

If

$$2x - 1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$\Rightarrow f\left(\frac{1}{2}\right) = 0$$

$$\begin{aligned} & 2\left(\frac{1}{2}\right)^3 + 7\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 3 \\ &= \frac{1}{4} + \frac{7}{4} + 1 - 3 \\ &= \frac{8}{4} + 1 - 3 \\ &= 3 - 3 \\ &= 0 \quad \text{QED} \end{aligned}$$

7. Investigate if $(2x + 1)$ is a factor of $2x^3 - x^2 - 5x - 2$.

Soln
 $f(-\frac{1}{2}) = 0$

$$\text{If } 2x + 1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$2\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 - 5\left(-\frac{1}{2}\right) - 2$$

$$= -\frac{1}{4} - \frac{1}{4} + \frac{5}{2} - 2$$

$$= -\frac{2}{4} + \frac{5}{2} - 2$$

$$= -\frac{2}{4} + \frac{10}{4} - \frac{8}{4} = 0$$