

Example 1

Complete the square on each of the following quadratic expressions.
Hence find the minimum value of each expression.

(i) $x^2 - 8x + 10$ (ii) $4x^2 + 4x + 2$

<p>Vertex form?</p> <p>Related perfect square?</p> $(a+b)^2 = a^2 + 2ab + b^2$ $a = x$ $b = \frac{1}{2}$ $b^2 = \frac{1}{4}$ $(x + \frac{1}{2})^2 = x^2 + 1x + \frac{1}{4}$	$4x^2 + 4x + 2$ $4[x^2 + 1x + \frac{1}{2}]$ $4[x^2 + 1x + \frac{1}{4} - \frac{1}{4} + \frac{1}{2}]$ $4[(x + \frac{1}{2})^2 + \frac{1}{4}]$ $4(x + \frac{1}{2})^2 + 1$ <p>min: $(-\frac{1}{2}, 1)$</p>
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Exercise 2.6

1. Find the value of c that completes the square in each of the following:

(i) $a^2 + 28a + c$

(ii) $x^2 - 6x + c$

(iii) $y^2 - 5y + c$

<p>$(a+b)^2 = a^2 + 2ab + b^2$</p> <p>(i)</p> $a = a$ $b = 14$ $b^2 = 196$	$a^2 + 28a + c$ $a^2 + 28a + 196$ $(a+14)(a+14)$
<p>(ii)</p> $a = x$ $b = -3$ $b^2 = 9$	$x^2 - 6x + c$ $x^2 - 6x + 9$ $(x-3)(x-3)$
<p>(iii)</p> $a = y$ $b = -\frac{5}{2}$ $b^2 = -\frac{25}{4}$	$y^2 - 5y + c$ $(y - \frac{5}{2})(y - \frac{5}{2})$ $y^2 - 5y - \frac{25}{4}$

2. Complete the square in each of the following: Vertex form?

(i) $x^2 - 8x - 3$ ~~20~~

(ii) $x^2 - 2x - 5$ ~~20~~

(iii) $x^2 - 2x + 1$ ~~20~~

<p><u>Formula method</u></p> <p>(i) $(a+b)^2 = a^2 + 2ab + b^2$</p> <p><u>Related perfect square?</u></p> <p>$a = x$ $b = -4$ $b^2 = 16$</p> <p>$x^2 - 8x + 16 = (x-4)^2$</p>	<p>$x^2 - 8x - 3$</p> <p>$x^2 - 8x + 16 - 16 - 3$</p> <p>$(x-4)^2 - 19$</p>
<p>(ii) <u>Related perfect square?</u></p> <p>$a = x$ $b = -1$ $b^2 = 1$</p> <p>$(x-1)^2 = x^2 - 2x + 1$</p>	<p>$x^2 - 2x - 5$</p> <p>$x^2 - 2x + 1 - 1 - 5$</p> <p>$(x-1)^2 - 6$</p>

2. Complete the square in each of the following: Vertex form

(i) $x^2 - 8x - 3$ ~~20~~

(ii) $x^2 - 2x - 5$ ~~20~~

(iii) $x^2 - 2x + 1$ ~~20~~

<p><u>Window method</u></p> <p><u>Related perfect square</u></p> <table border="1" style="margin-left: 20px;"> <tr> <td></td> <td style="text-align: center;">x</td> <td style="text-align: center;">-4</td> </tr> <tr> <td style="text-align: center;">x</td> <td style="text-align: center;">x^2</td> <td style="text-align: center;">$-4x$</td> </tr> <tr> <td style="text-align: center;">-4</td> <td style="text-align: center;">$-4x$</td> <td style="text-align: center;">$+16$</td> </tr> </table> <p>$(x-4)^2 = x^2 - 8x + 16$</p>		x	-4	x	x^2	$-4x$	-4	$-4x$	$+16$	<p>$x^2 - 8x - 3$</p> <p>$x^2 - 8x + 16 - 16 - 3$</p> <p>$(x-4)^2 - 19$</p>
	x	-4								
x	x^2	$-4x$								
-4	$-4x$	$+16$								