**4.** The graph of $y = a(x - p)^2 + q$ has a minimum point $(p, q)$.
By completing the square, find the minimum point of each of the following quadratic equations:

(i) $2x^2 + 4x - 5$ ~~= 0~~ (ii) $3x^2 - 6x - 1$ ~~= 0~~ (iii) $4x^2 + x + 3$ ~~= 0~~

$$f(x) = 2x^2 + 4x - 5 = 2\left[x^2 + 2x - \frac{5}{2}\right]$$

$$= 2\left[x^2 + 2x + 1 - 1 - \frac{5}{2}\right]$$

$$= 2\left[(x+1)^2 - \frac{7}{2}\right]$$

$$= 2(x+1)^2 - 7$$

| | x | 1 |
|---|---|---|
| x | $x^2$ | x |
| 1 | x | 1 |

change sign → same

min : $(-1, -7)$

**4.** The graph of $y = a(x - p)^2 + q$ has a minimum point $(p, q)$.
By completing the square, find the minimum point of each of the following quadratic equations:

(i) $2x^2 + 4x - 5$ ~~= 0~~ (ii) $3x^2 - 6x - 1$ ~~= 0~~ (iii) $4x^2 + x + 3$ ~~= 0~~

$$f(x) = 3x^2 - 6x - 1 = 3\left[x^2 - 2x - \frac{1}{3}\right]$$

$$= 3\left[x^2 - 2x + 1 - 1 - \frac{1}{3}\right]$$

$$= 3\left[(x-1)^2 - \frac{4}{3}\right]$$

$$= 3(x-1)^2 - 4$$

| | x | −1 |
|---|---|---|
| x | $x^2$ | −1x |
| −1 | −1x | +1 |

min : $(1, -4)$

**4.** The graph of $y = a(x - p)^2 + q$ has a minimum point $(p, q)$.
By completing the square, find the minimum point of each of the following quadratic equations:

(i) $2x^2 + 4x - 5 = 0$ (ii) $3x^2 - 6x - 1 = 0$ (iii) $4x^2 + x + 3 = 0$

$$f(x) = 4x^2 + x + 3 = 4\left[x^2 + \frac{1}{4}x + \frac{3}{4}\right]$$

| | $x$ | $\frac{1}{8}$ |
|---|---|---|
| $x$ | $x^2$ | $\frac{1}{8}x$ |
| $\frac{1}{8}$ | $\frac{1}{8}x$ | $\frac{1}{64}$ |

$$= 4\left[x^2 + \frac{1}{4}x + \frac{1}{64} - \frac{1}{64} + \frac{3}{4}\right]$$

$$= 4\left[\left(x + \frac{1}{8}\right)^2 + \frac{47}{64}\right]$$

$$= 4\left(x + \frac{1}{8}\right)^2 + \frac{47}{16}$$

min: $\left(-\frac{1}{8}, \frac{47}{16}\right)$