chapter 2 **Algebra 2**

## Section 2.4 Quadratic and linear equations in context



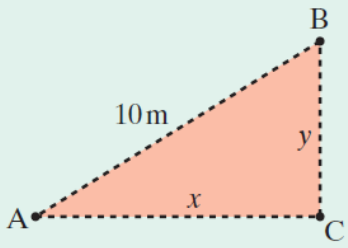
### Example 1

A right-angled triangle is to be made from a rope 24 m long. If the hypotenuse of the triangle, AB, has to be 10 m, find

(i) an equation in terms of $x$ and $y$ for the perimeter of the triangle

(ii) an equation in terms of $x$ and $y$ for the hypotenuse of the triangle.

(iii) Solve the equations to find possible lengths of the base ($x$) and height ($y$) of the triangle.




Perimeter = 24 $\Rightarrow$ $x + y + 10 = 24$

$\Rightarrow$ $x + y = 14$ (1)

Pytagoras $\Rightarrow$ $x^2 + y^2 = 10^2$

$\Rightarrow$ $x^2 + y^2 = 100$ (2)

and solve . . . .

## Exercise 2.4

**1.** Find the values of two consecutive numbers, the sum of the squares of which equals 61.

let $y$ = small no.
$x$ = big no.

$x - y = 1$ ①

$x^2 + y^2 = 61$ ②

① Rewrite linear

② Sub & solve

$(a+b)^2 = a^2 + 2ab + b^2$

$x = y + 1$

$(y+1)^2 + y^2 = 61$

$y^2 + 2y + 1 + y^2 = 61$

$2y^2 + 2y - 60 = 0$

$y^2 + y - 30 = 0$

$(y+6)(y-5) = 0$

$y = -6$ or $y = 5$

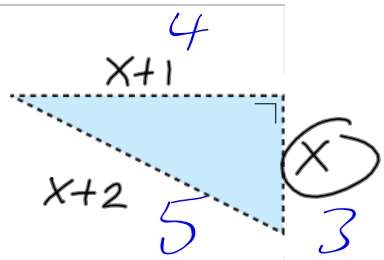
③ Sub back $x = y + 1$

$x = -6 + 1 = -5$ pt $(-5, -6)$

$x = 5 + 1 = 6$ pt $(6, 5)$

**4.** A right-angled triangle is to be made using three consecutive integer numbers as sides.

Find the length of the perimeter of the triangle.




$(x+2)^2 = (x+1)^2 + x^2$

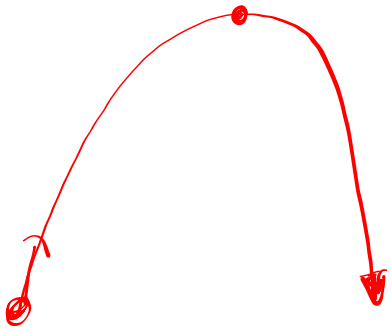
$x^2 + 4x + 4 = x^2 + 2x + 1 + x^2$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ ✓ or $x = -1$ no sense

**7.** A football is kicked up into the air. The height of the ball can be modelled by the equation $h = -16t^2 + 24t + 1$, where $h$ = the height in metres and $t$ = time in seconds.
At what times will the ball be at a height of 6 m?



$h = 6$

$6 = -16t^2 + 24t + 1$

$16t^2 - 24t + 5 = 0$

$(4t - 1)(4t - 5) = 0$ (−4t, −20t)

$4t - 1 = 0$
$4t = 1$
$t = \frac{1}{4}$

$4t - 5 = 0$
$4t = 5$
$t = \frac{5}{4} = 1\frac{1}{4}$

**12.** Find three consecutive integers such that three times their sum equals the product of the larger two.

let integers be $x, x+1, x+2$

$3[x + (x+1) + (x+2)] = (x+1)(x+2)$

expand: $3[3x+3] = x^2 + 2x + x + 2$

simplify: $9x + 9 = x^2 + 3x + 2$

$-9x, -9$: $x^2 - 6x - 7 = 0$

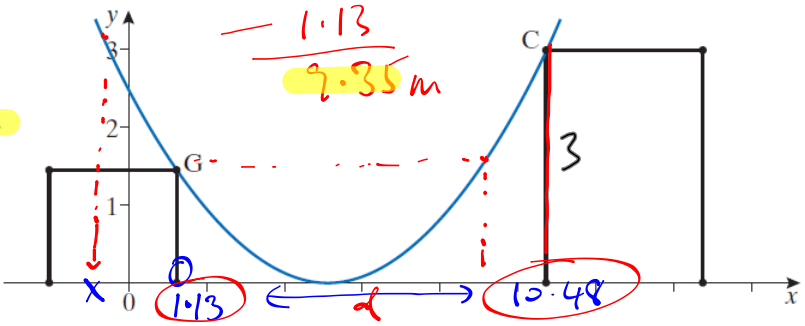
$(x + 1)(x - 7) = 0$

$x = -1$ or $7$

if $x = 1 \Rightarrow$ nos are: $-1, 0, 1$

if $x = 7 \Rightarrow$ no.s are: $7, 8, 9$

**15.** A skateboard ramp is in the shape of a curve with equation $h = 0.1x^2 - x + 2.5$. Two platforms represent the starting and finishing points as shown.




If the starting point C is at a height of 3 m and G, the finishing point, is at a height of 1.5 m, calculate the distance between the bases of the two platforms, correct to two places of decimals.

10.48 − 1.13 = 9.35 m

$1.5 = 0.1x^2 - x + 2.5$

$0.1x^2 - x + 1.0 = 0$

$x^2 - 10x + 10 = 0$

$a = 1$, $b = -10$, $c = 10$

$$x = \frac{10 \pm \sqrt{100 - 4(1)(10)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{60}}{2} = 1.13$$

$3 = 0.1x^2 - x + 2.5$

$0.1x^2 - x - 0.5 = 0$

$x^2 - 10x - 5 = 0$

$a = 1$ $b = -10$ $c = -5$

$$x = \frac{10 + \sqrt{(100 - 4(1)(-5)}}{2(1)}$$

$$= \frac{10 + \sqrt{120}}{2} = 10.48$$