Solving quadratic equations

(ii) Quadratic formula

If $ax^2 + bx + c = 0$,

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 2

Solve $x - 6 = \dfrac{3}{x}$. (Note: It is not always obvious that we are dealing with an equation of the form $ax^2 + bx + c = 0$.)

Multiply by LCD
ie. x
Rewrite is form:
$ax^2 + bx + c = 0$

$a = 1$
$b = -6$
$c = -3$

$x^2 - 6x = 3$

$x^2 - 6x - 3 = 0$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-3)}}{2(1)}$

$= \dfrac{6 \pm \sqrt{36+12}}{2} = \dfrac{6 \pm \sqrt{48}}{2} = \dfrac{6 \pm 4\sqrt{3}}{2}$

$x_1 = 3 - 2\sqrt{3}$

$x_2 = 3 + 2\sqrt{3}$

Solving quadratic equations (iv) Substitution

Example 3

Solve $x^4 + x^2 - 6 = 0$ for $x \in R$.

Degree 4
treat like quadratic

$x^4 + x^2 - 6 = 0$

$(x^2 + 3)(x^2 - 2) = 0$

then solve resulting 2 quadratics

$x^2 + 3 = 0$ | $x^2 - 2 = 0$

$x^2 = -3$ | $x^2 = 2$

$x = \pm\sqrt{-3}$ | $x = \pm\sqrt{2}$

PROBLEM!

Degree 4
$\Rightarrow$ 4 solutions

Example 4

Solve $2x + 3\sqrt{x} = 5$ for $x \in R$.

Isolate $\sqrt{}$

$-3\sqrt{x}, -5$

$(2x-5)^2 = (-3\sqrt{x})^2$

Square

$(a+b)^2 = a^2+2ab+b^2$

$4x^2 - 20x + 25 = 9x$

$-9x$

$4x^2 - 29x + 25 = 0$

$(4x - 25)(x - 1) = 0$

$-4x$
$-25x$
$-29x$

$4x - 25 = 0$ | $x = 1$

$4x = 25$

$x = \frac{25}{4}$

6. Solve $2x^2 - \sqrt{3}x - 3 = 0$.

use

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-\sqrt{3}) \pm \sqrt{(-\sqrt{3})^2 - 4(2)(-3)}}{2(2)}$

$a = 2$
$b = -\sqrt{3}$
$c = -3$

$= \frac{+\sqrt{3} \pm \sqrt{3+24}}{4}$

$= \frac{\sqrt{3} \pm \sqrt{27}}{4}$

could use calculator

$= \frac{\sqrt{3} \pm 3\sqrt{3}}{4}$

$x_1 = \frac{\sqrt{3} - 3\sqrt{3}}{4} = -\frac{2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$

$x_2 = \frac{\sqrt{3} + 3\sqrt{3}}{4} = \frac{4\sqrt{3}}{4} = \sqrt{3}$