Algebraic identities and factors

Example 4

Given that $(x - t)^2$ is a factor of $x^3 + 3px + c$, show that $p = -t^2$ and $c = 2t^3$.

CUBIC EXPRESSION

expand $(x-t)^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$(x-t)^2 = x^2 - 2xt + t^2$

QUADRATIC FACTOR

FACTOR DIVIDES WITH NO REMAINDER

D M S A

LINEAR

$x + 2t$

$x^2 - 2tx + t^2 \,)\, x^3 + 0x^2 + 3px + c$

$\mp x^3 \pm 2tx^2 \mp t^2x$

$2tx^2 + (3p - t^2)x + c$

$\mp 2tx^2 \pm (4t^2)x \mp 2t^3$

$0x + 0$

$(3p - t^2) + 4t^2 = 0$

Simplify $\div 3$

$3p + 3t^2 = 0$

$p + t^2 = 0$

$p = -t^2$ (1)

QED

$c - 2t^3 = 0$

$c = 2t^3$ (2)

QED

2. If $(3x - 2)(x + 5) = 3x^2 + px + q$ for all values of x, find the values of p and q.

RHS LHS

expand RHS

$3x^2 + 15x - 2x - 10 = 3x^2 + px + q$

$3x^2 + 13x - 10 = 3x^2 + px + q$

Compare Coefficients

$13 = p$

$q = -10$

3. If $x^2 + 6x + 16 = (x + a)^2 + b$ for all values of x, find the values of a and b.

LHS RHS

expand RHS

$(a+b)^2 = a^2+2ab+b^2$

$x^2+6x+16 = [x^2+2ax+a^2]+b$

$1x^2+6x+16 = 1x^2+2ax+a^2+b$

compare coefficients

$6=2a$

$\boxed{a=3}$ (1)

$16=a^2+b$ (2)

Sub (1) into (2)

$16=(3)^2+b$

$16=9+b$

$\boxed{7=b}$

4. Find the real numbers a and b such that $x^2 + 4x - 6 = (x + a)^2 + b$ for all $x \in R$.

6. Find the values of a and b if $(2x + a)^2 = 4x^2 + 12x + b$, for all x.