

19. Write $\frac{1}{(x+1)(x+4)}$ as the partial fractions $\frac{A}{(x+1)} + \frac{B}{(x+4)}$.

let $\frac{1}{(x+1)(x+4)} = \frac{A}{(x+1)} + \frac{B}{(x+4)}$

Box-tie $= \frac{A(x+4) + B(x+1)}{(x+1)(x+4)}$

equate the numerators $\Rightarrow 1 = A(x+4) + B(x+1)$

expand $1 = Ax + 4A + Bx + B$

Compare coefficients $0x + 1 = (A+B)x + 4A + B$

\Rightarrow ① $A + B = 0$
 $A = -B$

② $4A + B = 1$
 $\rightarrow 4(-B) + B = 1$
 $-3B = 1$
 $B = -\frac{1}{3}$

$\Rightarrow A = -(-\frac{1}{3}) = \frac{1}{3}$
 $A = \frac{1}{3}$

21. If $(x-2)^2$ is a factor of $x^3 + px + q$, find the value of p and the value of q .

expand
 $(a+b)^2 = a^2 + 2ab + b^2$

factors divide evenly \Rightarrow remainder = 0

$$(x-2)^2 = x^2 - 4x + 4$$

$$\begin{array}{r} x^3 + 0x^2 + px + q \\ \underline{-(x^2 - 4x + 4)} \\ 4x^2 + (p-4)x + q \\ \underline{-(4x^2 - 16x + 16)} \\ 0x + 0 \end{array}$$

\Rightarrow ① $p - 4 + 16 = 0$

$p = -12$

and \Rightarrow ② $q - 16 = 0$

$q = 16$