Example 5

$2x - \sqrt{3}$ is a factor of $4x^2 - 2(1 + \sqrt{3})x + \sqrt{3}$; find the second factor.

LINEAR   QUADRATIC

DIVIDE

$-2(1+\sqrt{3})x + 2\sqrt{3}x$
$= -2x - 2\sqrt{3}x + 2\sqrt{3}x$

$$\begin{array}{r} 2x - 1 \\ 2x - \sqrt{3}\overline{)\,4x^2 - 2(1+\sqrt{3})x + \sqrt{3}} \\ \underline{\mp 4x^2 \pm 2\sqrt{3}x} \\ -2x + \sqrt{3} \\ \underline{\pm 2x \mp \sqrt{3}} \end{array}$$

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20. If $(x-3)^2$ is a factor of $x^3 + ax + b$, find the value of a and the value of b.

$(x-3)^2$ divides into
cubic with no remainder

expand $(x-3)^2 = x^2 - 6x + 9$

Divide
D m s A

$$\begin{array}{r} x + 6 \\ x^2 - 6x + 9\overline{)\,x^3 + 0x^2 + ax + b} \\ \underline{\mp x^3 \pm 6x^2 \mp 9x} \\ 6x^2 + (a-9)x + b \\ \underline{\mp 6x^2 \pm 36x \mp 54} \\ 0x \quad + 6 \end{array}$$

Compare
coefficients

$a - 9 + 36 = 0$ | $b - 54 = 0$
$a = -27$ | $b = 54$

1

21. If $(x-2)^2$ is a factor of $x^3 + px + q$, find the value of p and the value of q.

Plan divide
quadratic into cubic
and Rem. = 0

expand
$(a+b)^2 = a^2 + 2ab + b^2$

$(x-2)^2 = x^2 - 4x + 4$

divide
D   m   S   A

$$\begin{array}{r} x + 4 \\ x^2-4x+4 \overline{)\; x^3 + 0x^2 + px + q} \\ \underline{\pm x^3 \pm 4x^2 \mp 4x} \\ 4x^2 + (p-4)x + q \\ \underline{\pm 4x^2 \pm 16x \mp 16} \\ 0x \quad + 0 \end{array}$$

Compare
Coefficients

$p - 4 + 16 = 0$ $q - 16 = 0$
$p = -12$ $q = 16$

23. If $(x^2 + b)$ is a factor of $x^3 - 3x^2 + bx - 15$, find the value of b.

Plan: divide
quadratic into
cubic ⇒ Rem. = 0

$x^2 + b = x^2 + 0x + b$

divide
D   M   S   A

$$\begin{array}{r} x - 3 \\ x^2+0x+b \overline{)\; x^3 - 3x^2 + bx - 15} \\ \underline{\pm x^3 \pm 0x^2 \mp bx} \\ -3x^2 + 0x - 15 \\ \underline{\pm 3x^2 \pm 0x \pm 3b} \\ 0 \end{array}$$

⇒ $-15 + 3b = 0$
$3b = 15$
$b = 5$

compare
cofficients

2