## 3. Perfect squares

### Example 3

Given that $25x^2 + px + 16$ is a perfect square and $p > 0$, find the value of $p$.

$(a+b)^2 = a^2+2ab+b^2$

$= 25x^2 + px + 16$

$a^2 = 25x^2 \Rightarrow a = 5x$

$b^2 = 16 \Rightarrow b = 4$

$+2ab = ?$

$= 2(5x)(4) = 40x$

perfect square

$25x^2 + 40x + 16$ ✓

factorise to check

$(5x+4)(5x+4)$ ✓

$p = 40$

### Example 4

Divide $(2x^3 - 11x + 6)$ by $(2x^2 + 4x - 3)$.

$x-2$

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$$\begin{array}{r} x - 2 \\ 2x^2 + 4x - 3 \overline{)\, 2x^3 + 0x^2 - 11x + 6} \\ \mp 2x^3 \mp 4x^2 \pm 3x \\ \hline -4x^2 - 8x + 6 \\ \pm 4x^2 \pm 8x \mp 6 \\ \hline \end{array}$$

**7.** Expand each of the following perfect squares.

$(a+b)^2 = a^2 + 2ab + b^2$

(i) $(x+2)^2 = x^2 + 4x + 4$

(ii) $(x-3)^2 = x^2 - 6x + 9$

(iii) $(x+5)^2 = x^2 + 10x + 25$

(iv) $(a+b)^2 = a^2 + 2ab + b^2$

(v) $(x-y)^2 = x^2 - 2xy + y^2$

(vi) $(a+2b)^2 = a^2 + 4ab + 4b^2$

(vii) $(3x-y)^2 = 9x^2 - 6xy + y^2$

(viii) $(x-5y)^2 = x^2 - 10xy + 25y^2$

(ix) $(2x+3y)^2 = 4x^2 + 12xy + 9y^2$

**10.** If $px^2 + 4x + 1$ is a perfect square for all values of $x$, find the value of $p$.

$(a+b)^2 = a^2 + 2ab + b^2$

$= px^2 + 4x + 1$

$\Rightarrow a^2 = px^2$ ①

by observation $2ab = 4x$ ②

$b^2 = 1 \Rightarrow b = 1$ ③

Sub ③ into ② $\Rightarrow 2a(1) = 4x$

$2a = 4x \Rightarrow a = 2x$ ④

Sub ④ into ① $(2x)^2 = px^2$

$4x^2 = px^2 \Rightarrow p = 4$

**21.** Simplify each of the following quotients:

(i) $\dfrac{6x^2y + 9xy^2 - 3xy}{3xy}$ (ii) $\dfrac{6x^4 - 9x^3 + 12x^2}{3x^2}$

Notice : $3xy$ is common factor of each term in numerator

$6x^2y = 3xy(2x)$

$9xy^2 = 3xy(3y)$

$-3xy = 3xy(-1)$

$$\frac{6x^2y + 9xy^2 - 3xy}{3xy} = 2x + 3y - 1$$

$$= 2x + 3y - 3$$

**25.** Perform the following operations:

long division

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(iii) $3x^3 + 2x^2 - 7x + 2 \div (x^2 + x - 2)$

$$\begin{array}{r} 3x - 1 \\ x^2 + x - 2 \overline{)\,3x^3 + 2x^2 - 7x + 2} \\ -3x^3 - 3x^2 + 6x \\ \hline -x^2 - x + 2 \\ +x^2 + x - 2 \\ \hline \end{array}$$

(iv) $5x^3 + 14x^2 + 7x - 2 \div (5x^2 + 4x - 1)$

$$\begin{array}{r} x + 2 \\ 5x^2 + 4x - 1 \overline{)\,5x^3 + 14x^2 + 7x - 2} \\ -5x^3 - 4x^2 + 1x \\ \hline 10x^2 + 8x - 2 \\ -10x^2 - 8x + 2 \\ \hline \end{array}$$