Statistics 2

Chapter 4

Section 4.3 The normal distribution

PROJECT MATHS Text & Tests 5
LEAVING CERTIFICATE
HIGHER LEVEL
STRAND 1
PROBABILITY & STATISTICS

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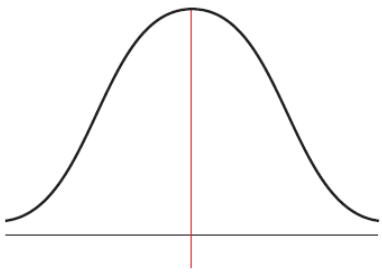
In Section 3.6 of this book, we were introduced to the **normal distribution** which is the cornerstone of modern statistics.

The **normal curve** is a smooth bell-shaped and symmetrical curve.
The red line is the axis of symmetry.

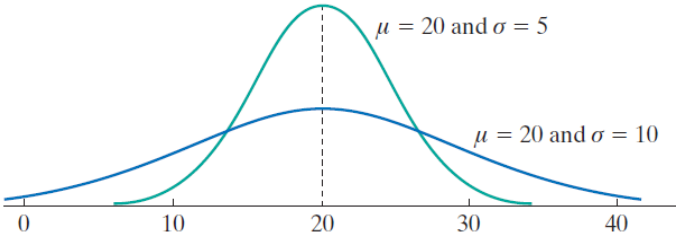


The mean, the mode and the median are all equal and they lie on the axis of symmetry.

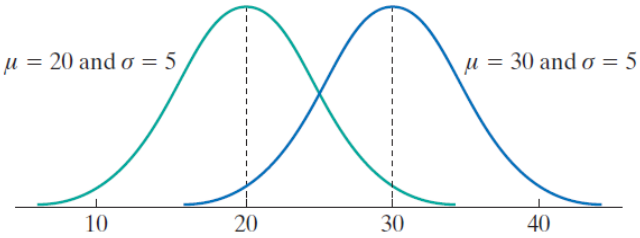
Normal distributions occur frequently in nature. For example the heights and weights of all adult males in Ireland will be **normally distributed**.
If all the heights of these adult males were plotted on a graph, the result would be a smooth bell-shaped curve, as shown above.

All normal distributions will have a mean (μ) and standard deviation (σ).
Different values for μ and σ will give different normal distributions.

The diagram on the right shows two normal distributions with the same mean but different standard deviations.



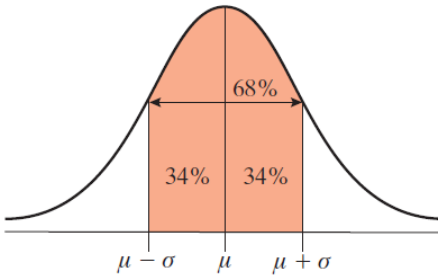
This diagram shows two normal distributions with the same standard deviation but different means.



All normal distributions share some very important characteristics.

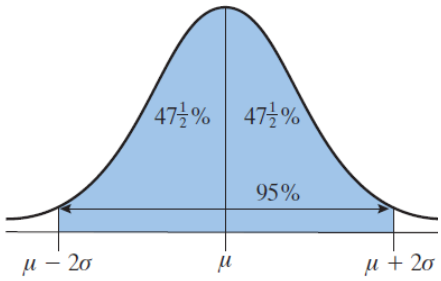
1. About 68% of all the values of any normal distribution lie within one standard deviation of the mean, i.e., in the range $[\mu - \sigma \text{ and } \mu + \sigma]$.

34% lie to the right of the mean.
34% lie to the left of the mean.

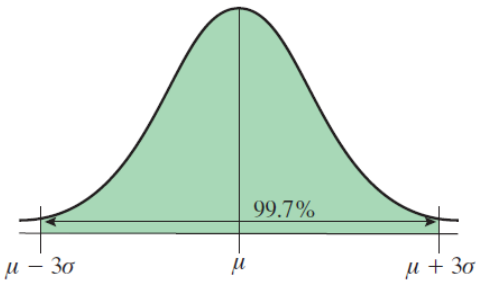


2. About 95% of all values lie within two standard deviations of the mean, i.e., in the range $[\mu - 2\sigma \text{ and } \mu + 2\sigma]$.

$47\frac{1}{2}\%$ lie to the right of the mean.
$47\frac{1}{2}\%$ lie to the left of the mean.



3. Almost all (99.7%) of the values lie within three standard deviations of the mean.



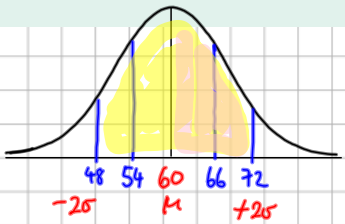
Example 1

The marks, out of 100, in an examination are normally distributed.
The mean mark is 60 and the standard deviation is 6 marks.

(i) Work out the mark that is two standard deviations above the mean.
(ii) What percentage of the marks lie between 48 and 72 marks?
(iii) What percentage of the marks lie between 60 and 72 marks?
(iv) If 1000 students took the examination, how many students scored less than 54 marks?

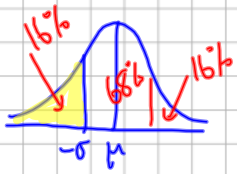
$\mu = 60$

$\sigma = 6$



(i) 2 standard deviations above mean
$= \mu + 2\sigma = 60 + 2(6) = 72$

(ii) between 48 and 72 is $\pm 2\sigma$
$= 95\%$ (Empirical Rule)



(iii) between 60 and 72 is $95\%/2 = 47.5\%$

(iv) 16% get less than 54
16% of 1000 = 160 people